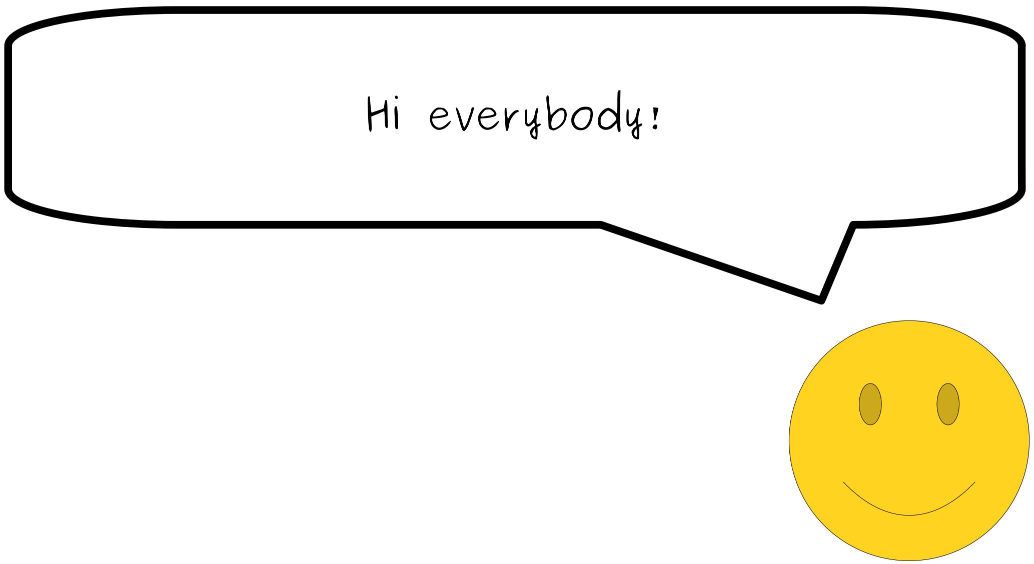
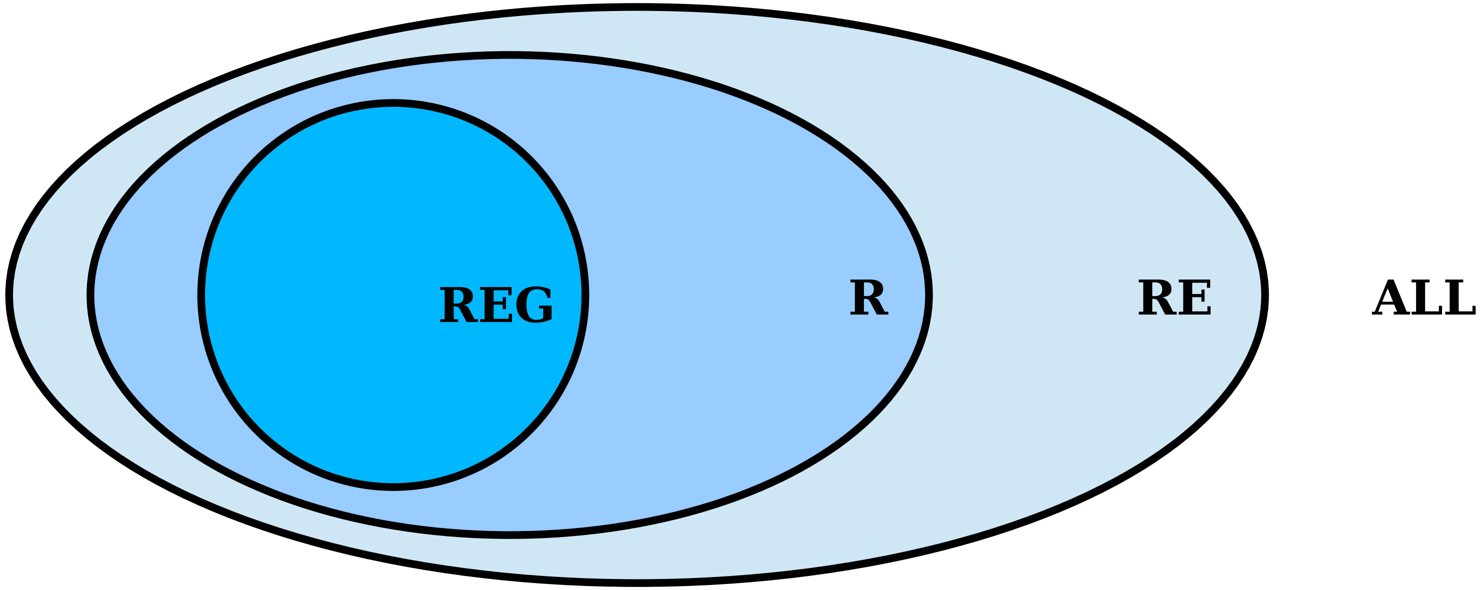
The Guide to the Lava Diagram


Hi everybody!

As you probably noticed on Problem Set Nine – we love asking questions about "The Lava Diagram."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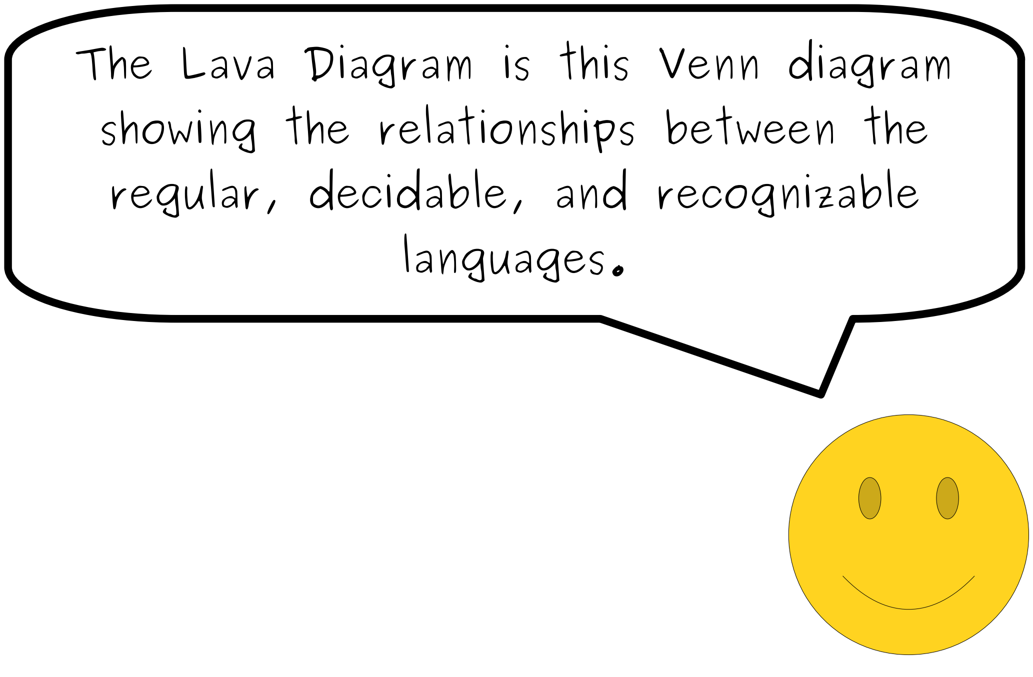
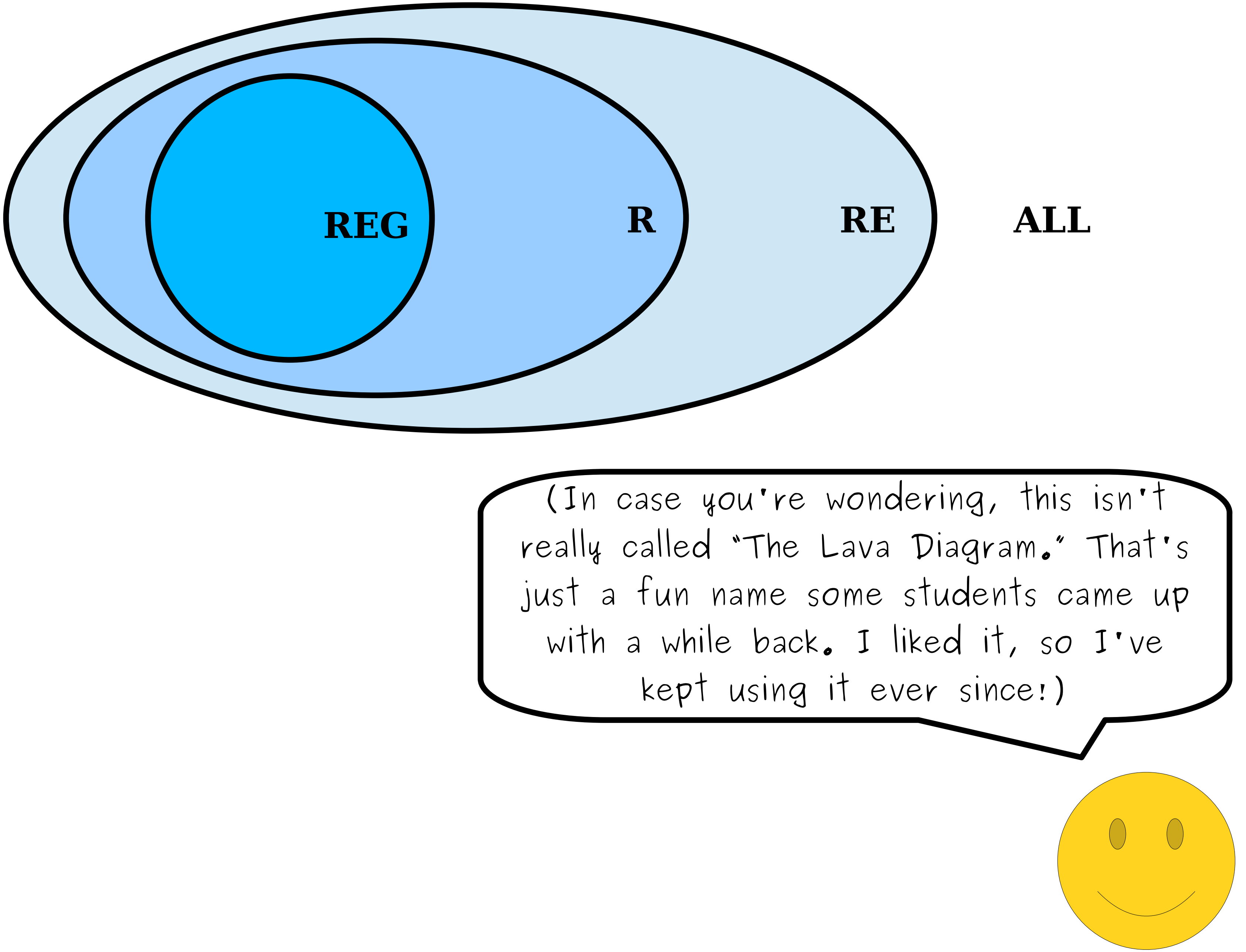

The Lava Diagram is this Venn diagram showing the relationships between the regular, decidable, and recognizable languages.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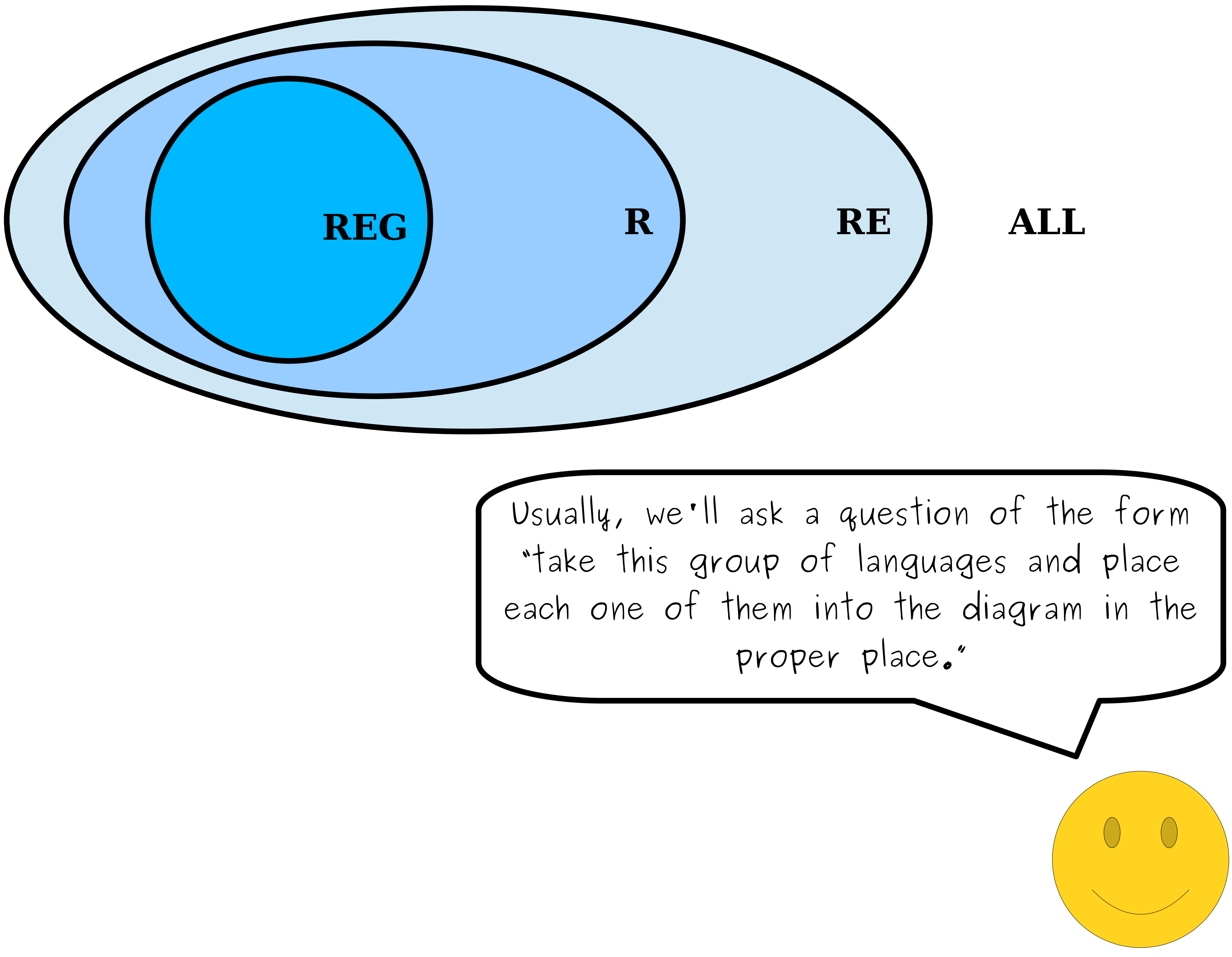
(In case you're wondering, this isn't really called "The Lava Diagram." That's just a fun name some students came up with a while back. I liked it, so I've kept using it ever since!)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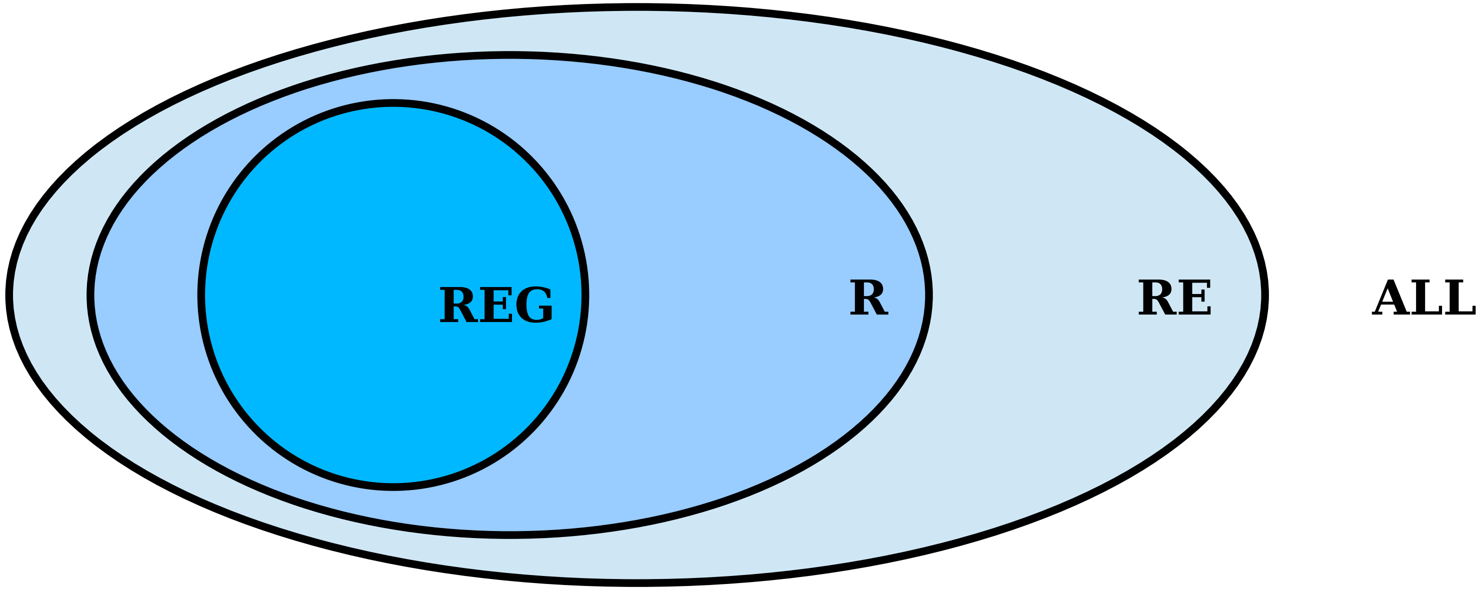
Usually, we'll ask a question of the form "take this group of languages and place each one of them into the diagram in the proper place."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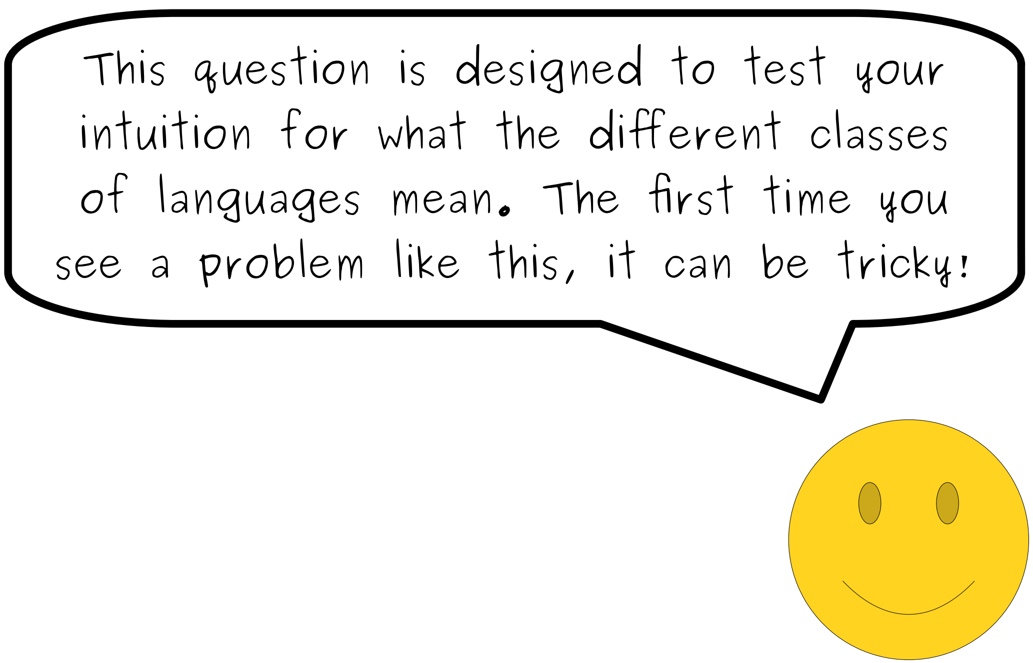
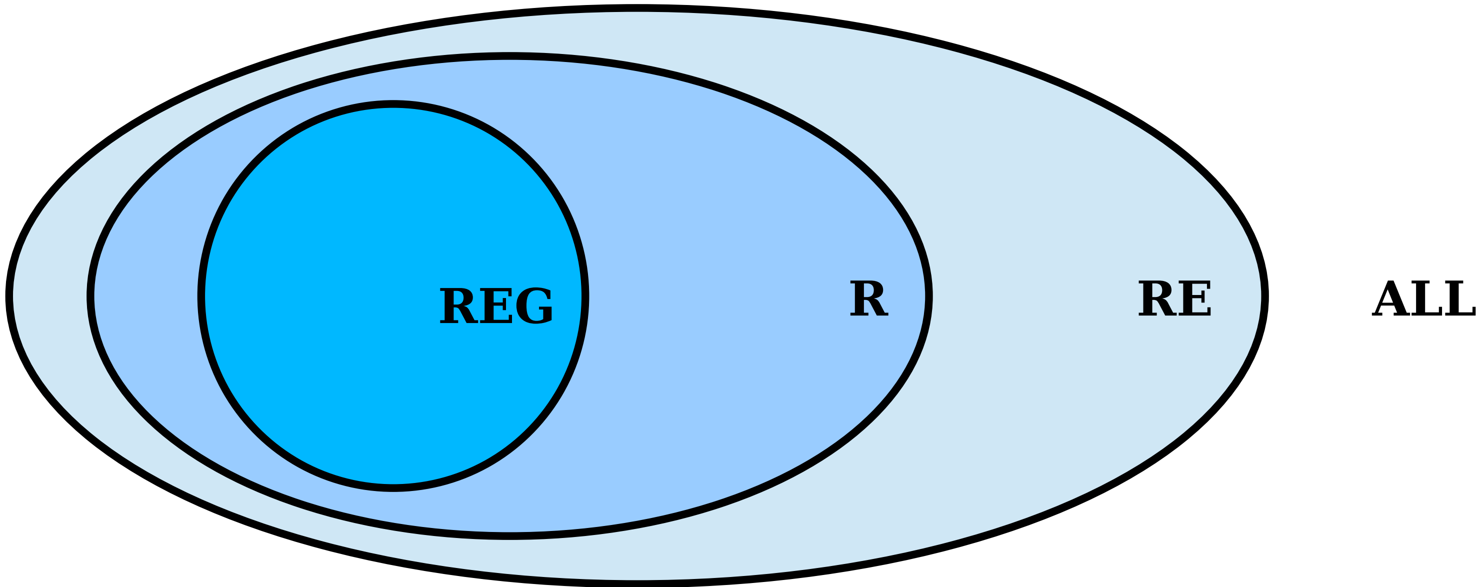
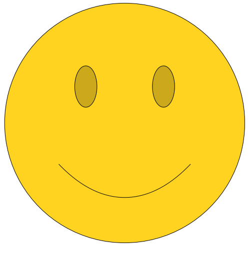

This question is designed to test your intuition for what the different classes of languages mean. The first time you see a problem like this, it can be tricky!



However, there are a bunch of useful intuitions that can help guide you while working on these problems. We'll go and talk about them by working through these four languages here.



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$L_2 = \{ \langle M \rangle \mid M$ is a TM that accepts exactly two strings $\}$

$L_3 = \{ \mathtt{a}^n\mathtt{b}^n \mid n \in \mathbb{N}$ and $n > 1000 \}$

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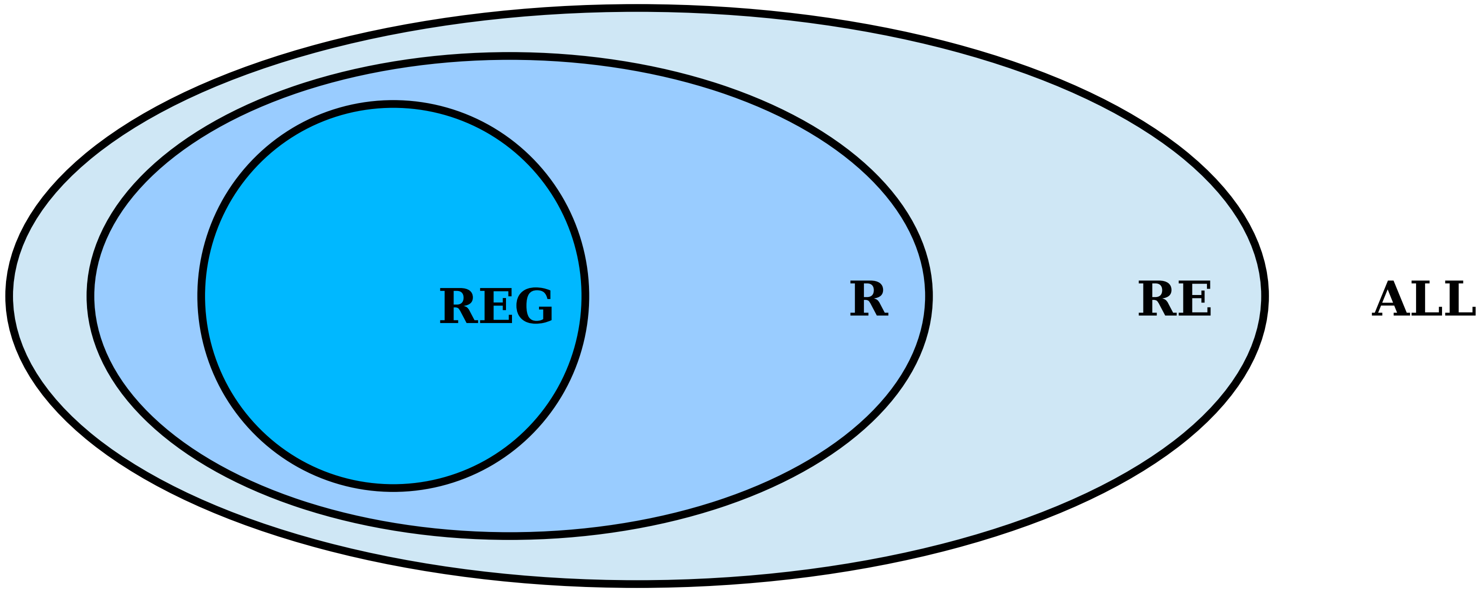

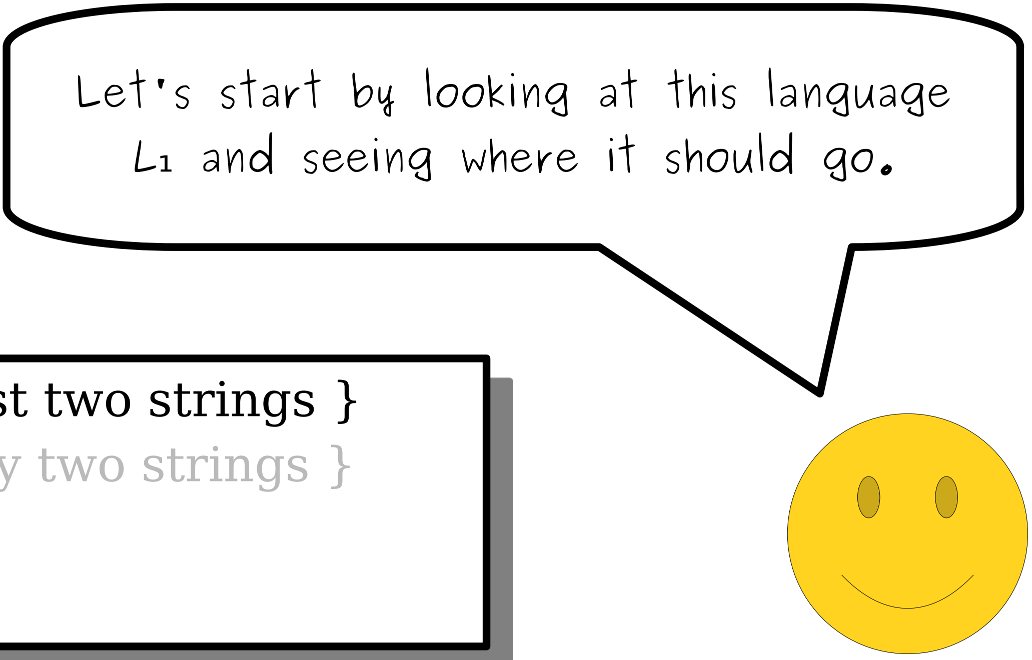

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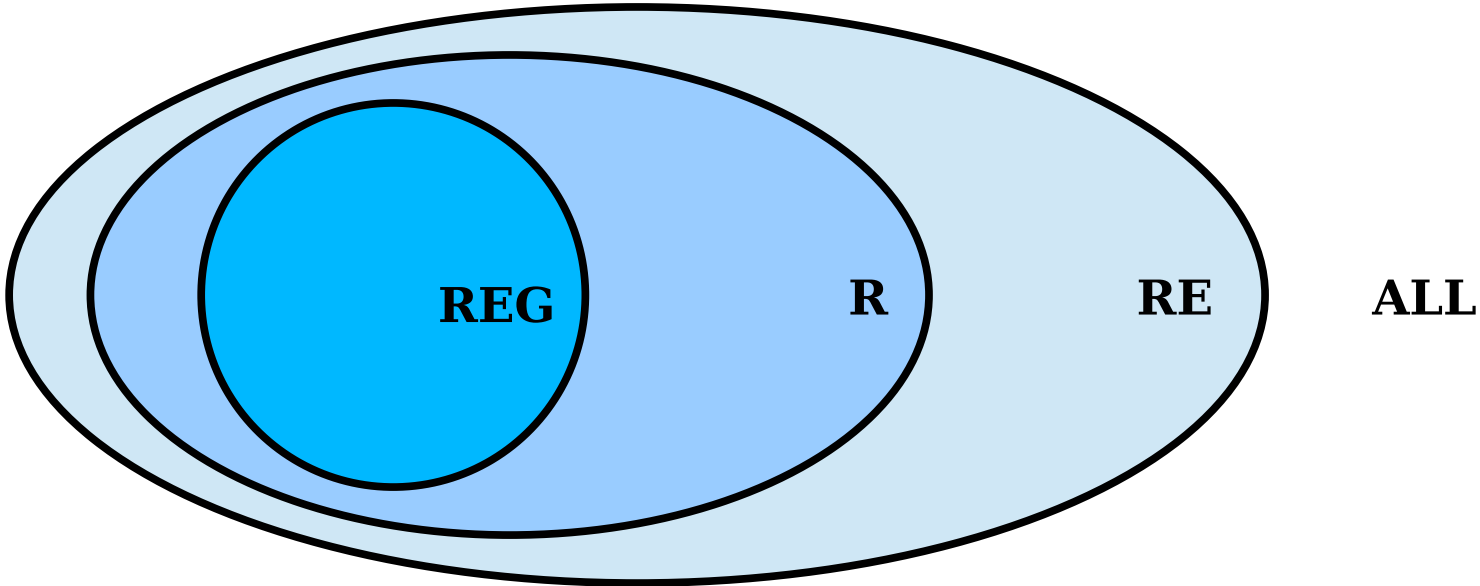

There are a couple of different strategies you can use to work through these problems, but the one we find the most useful is to start from the outside and work inward.

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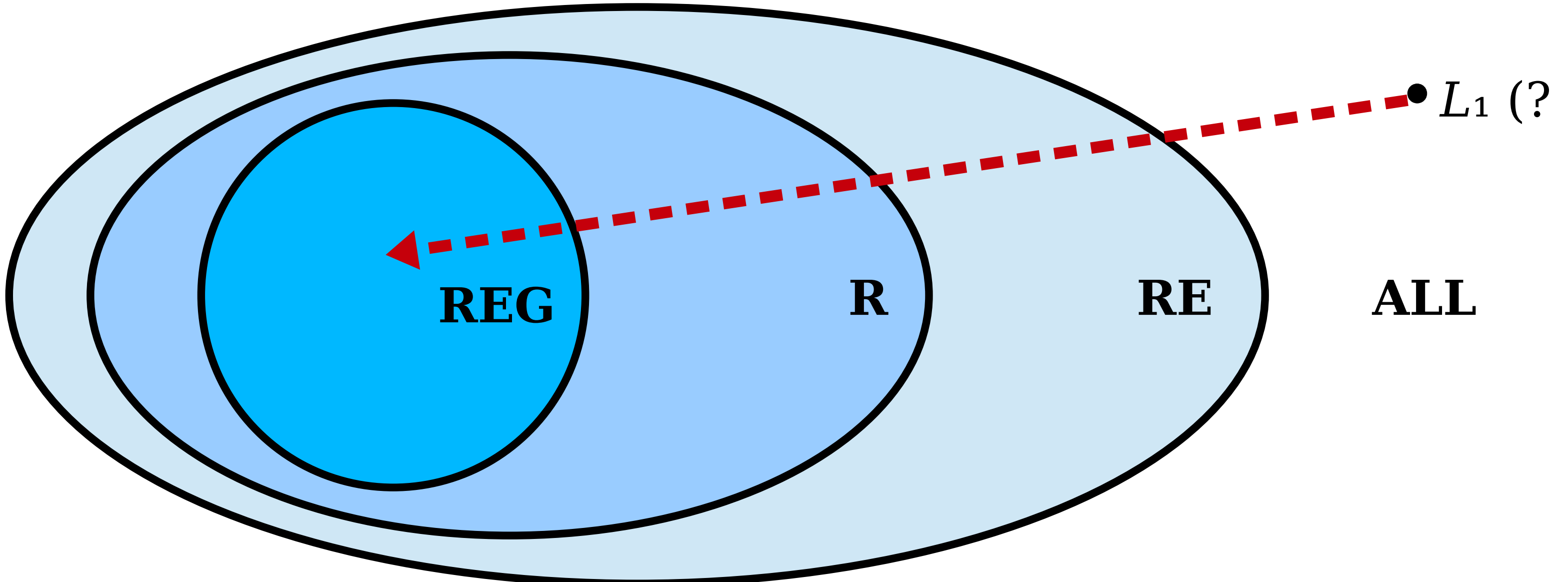
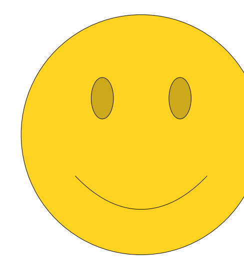


That is, we're going to start off with L_1 in the **ALL** section, then try to see how far down we can push it into the Lava Diagram.



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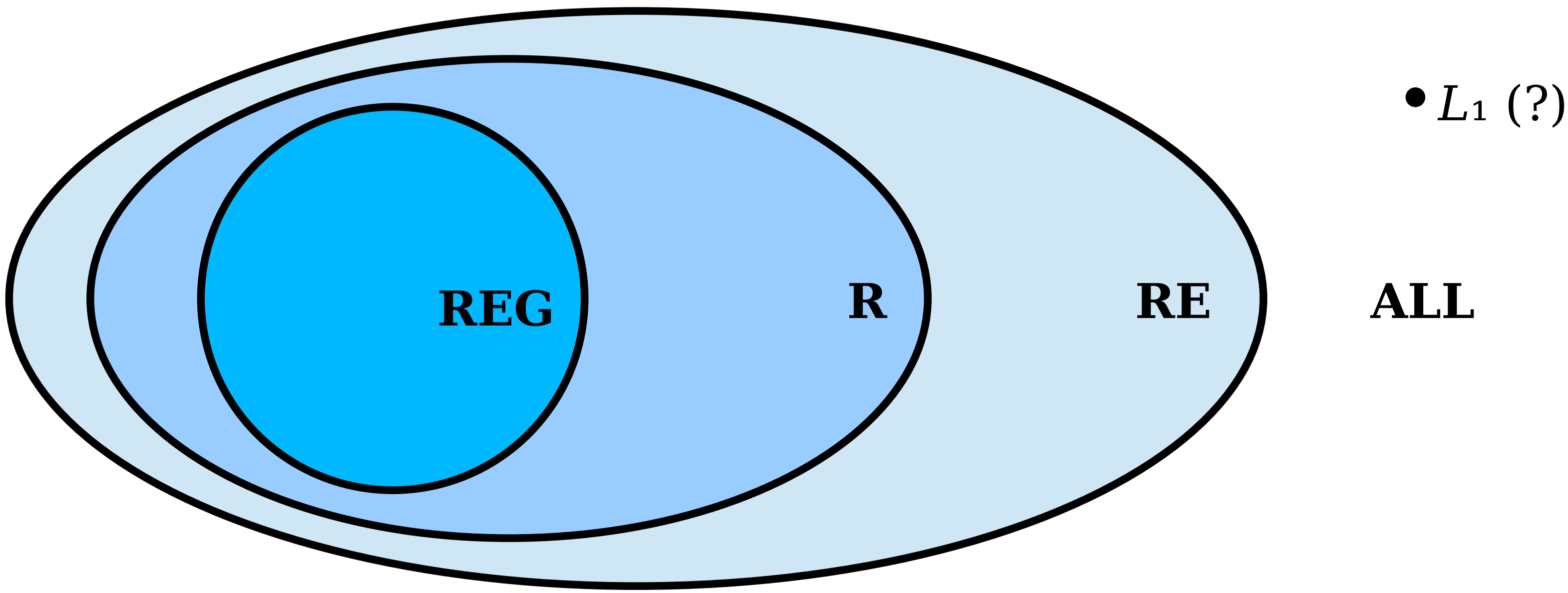

The very first question we should ask ourselves, therefore, is whether this language belongs to **RE**.

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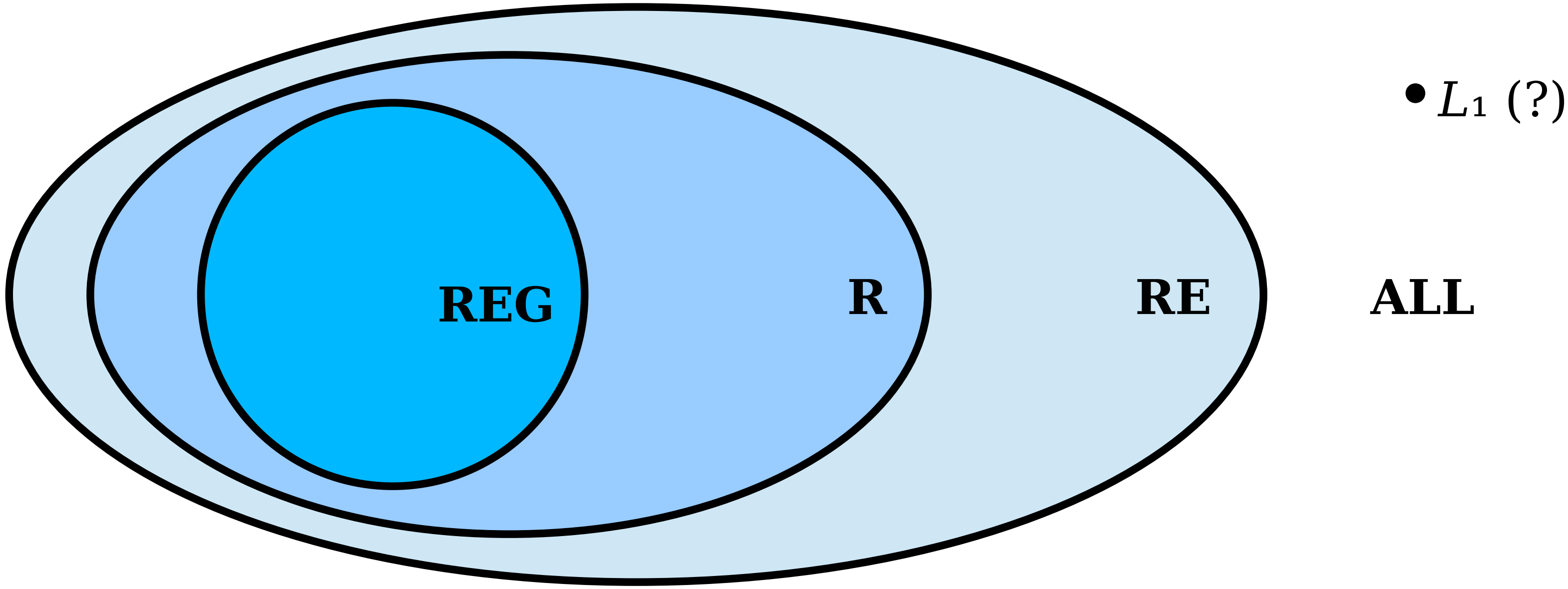
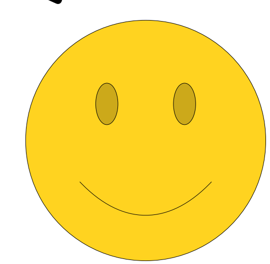


So what exactly is the class **RE**?



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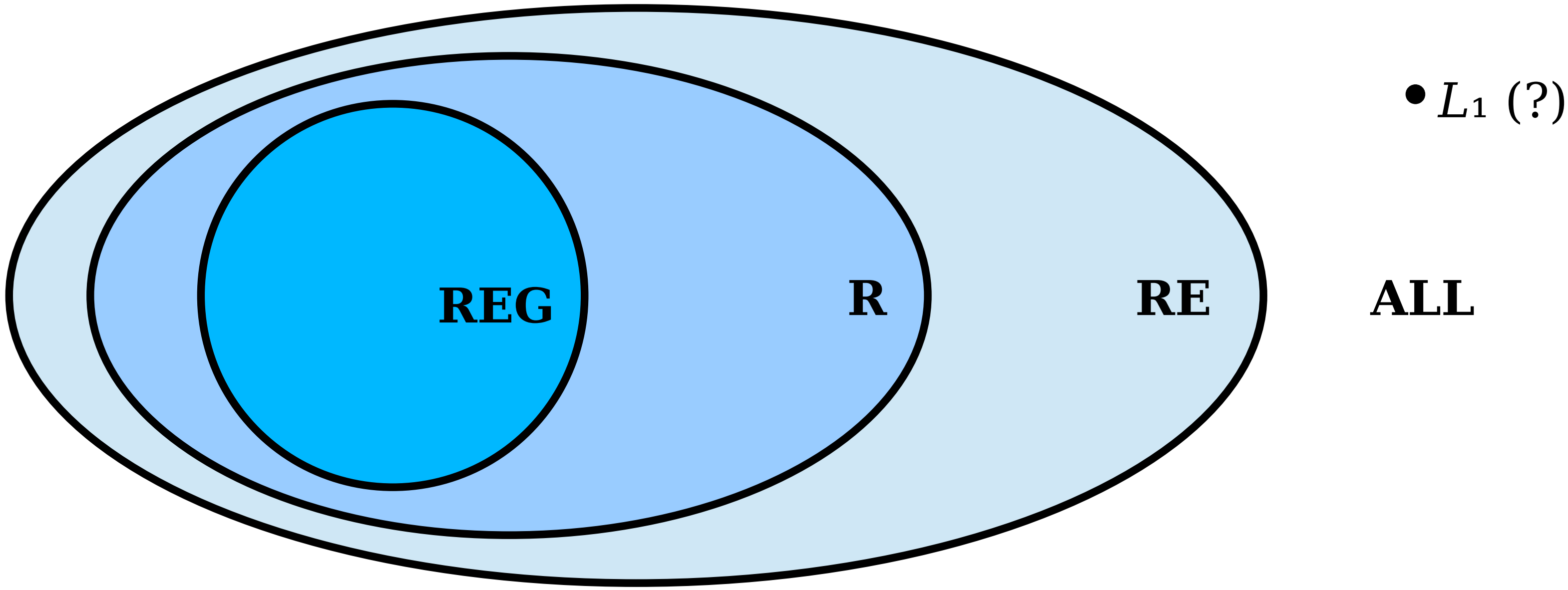

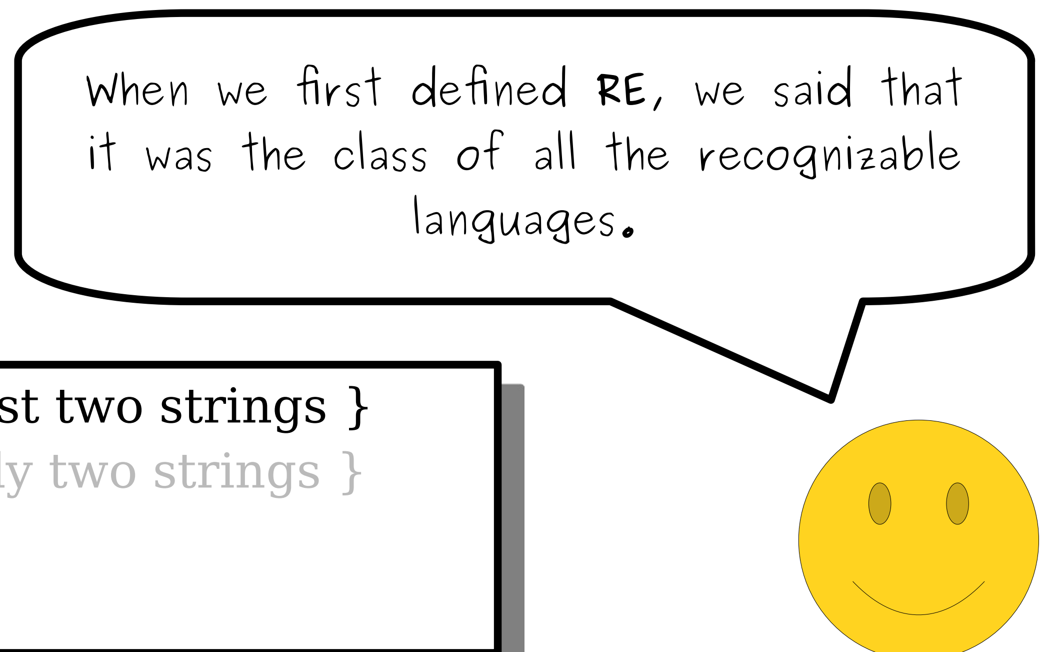

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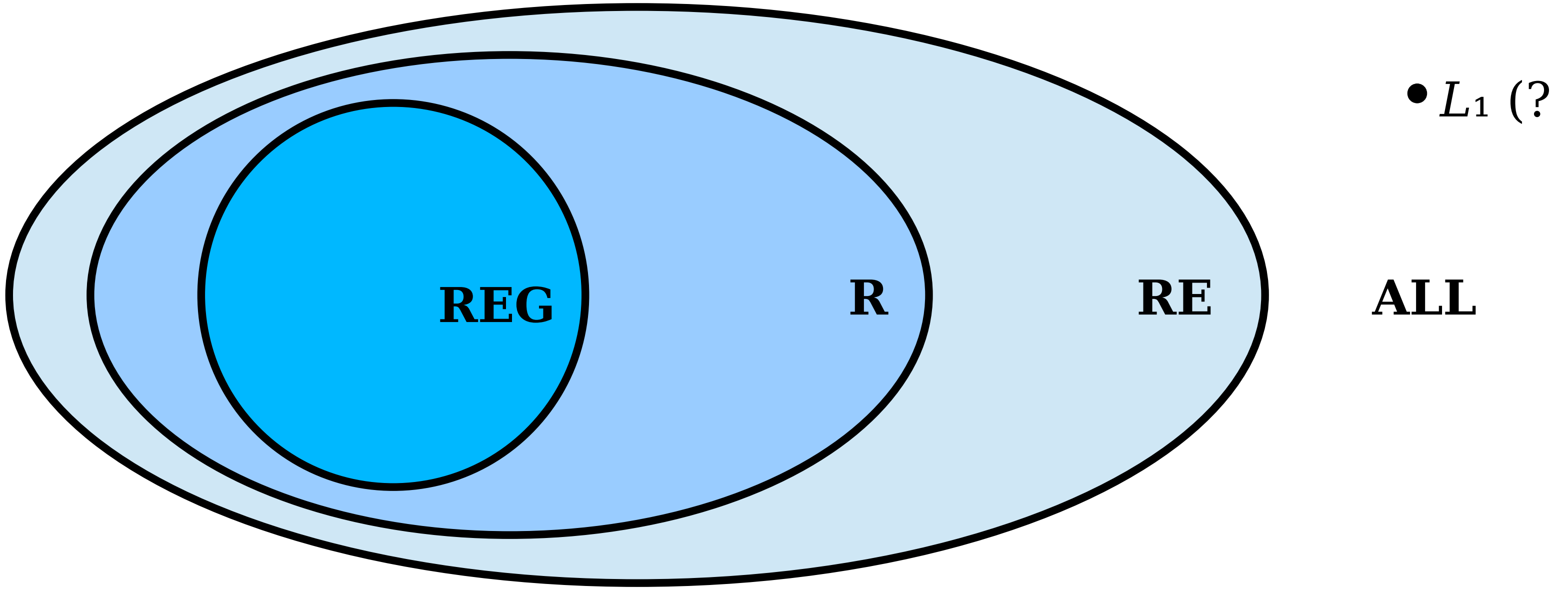
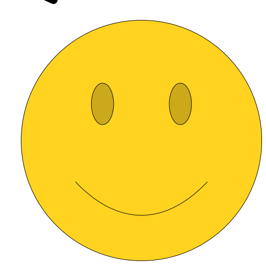


This means that we <u>could</u> try to think about **RE** as "the class of problems with recognizers."



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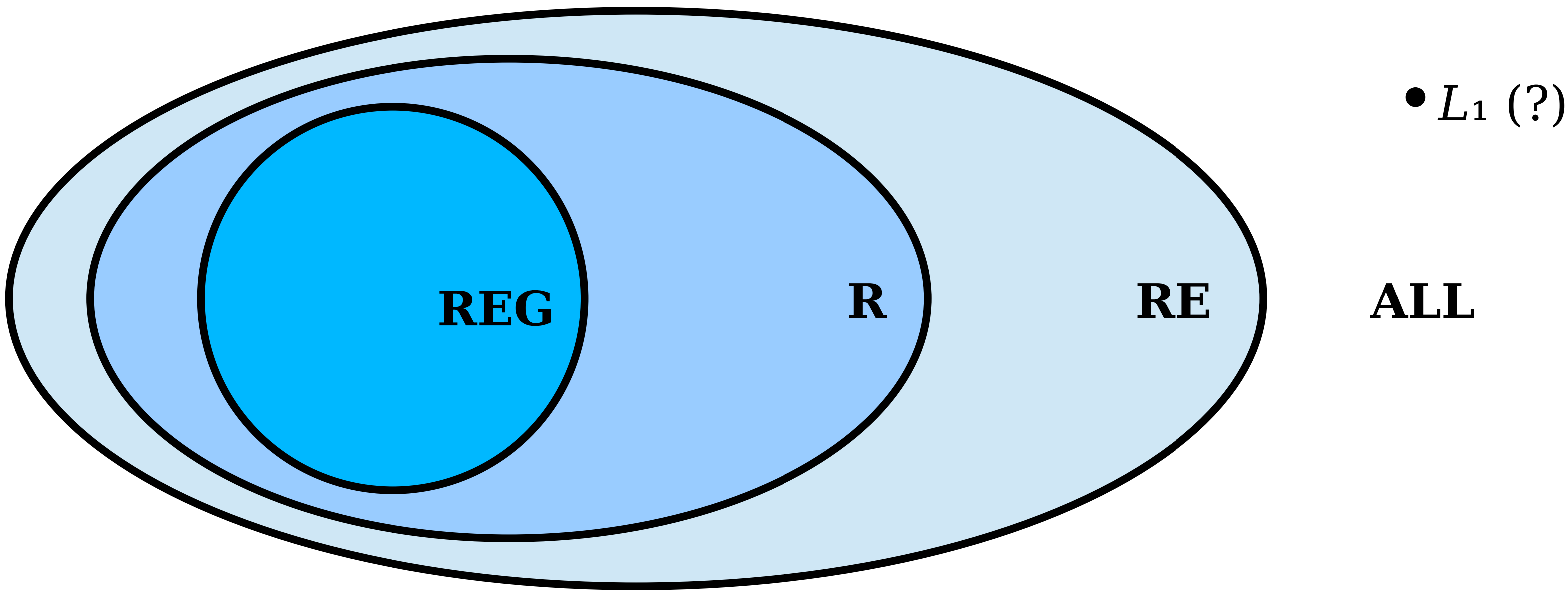

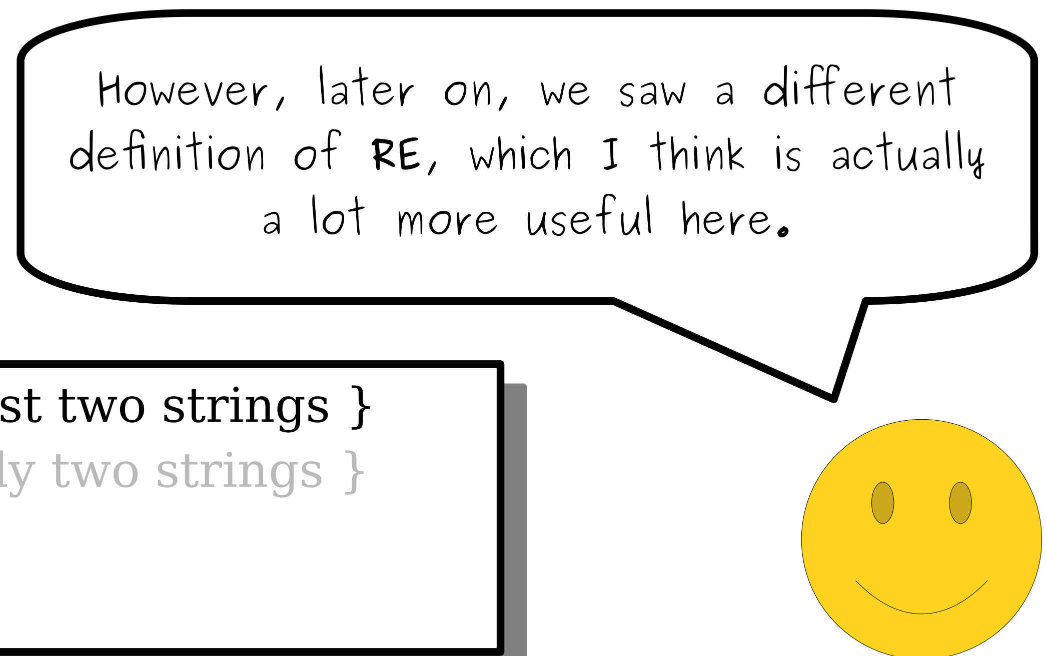

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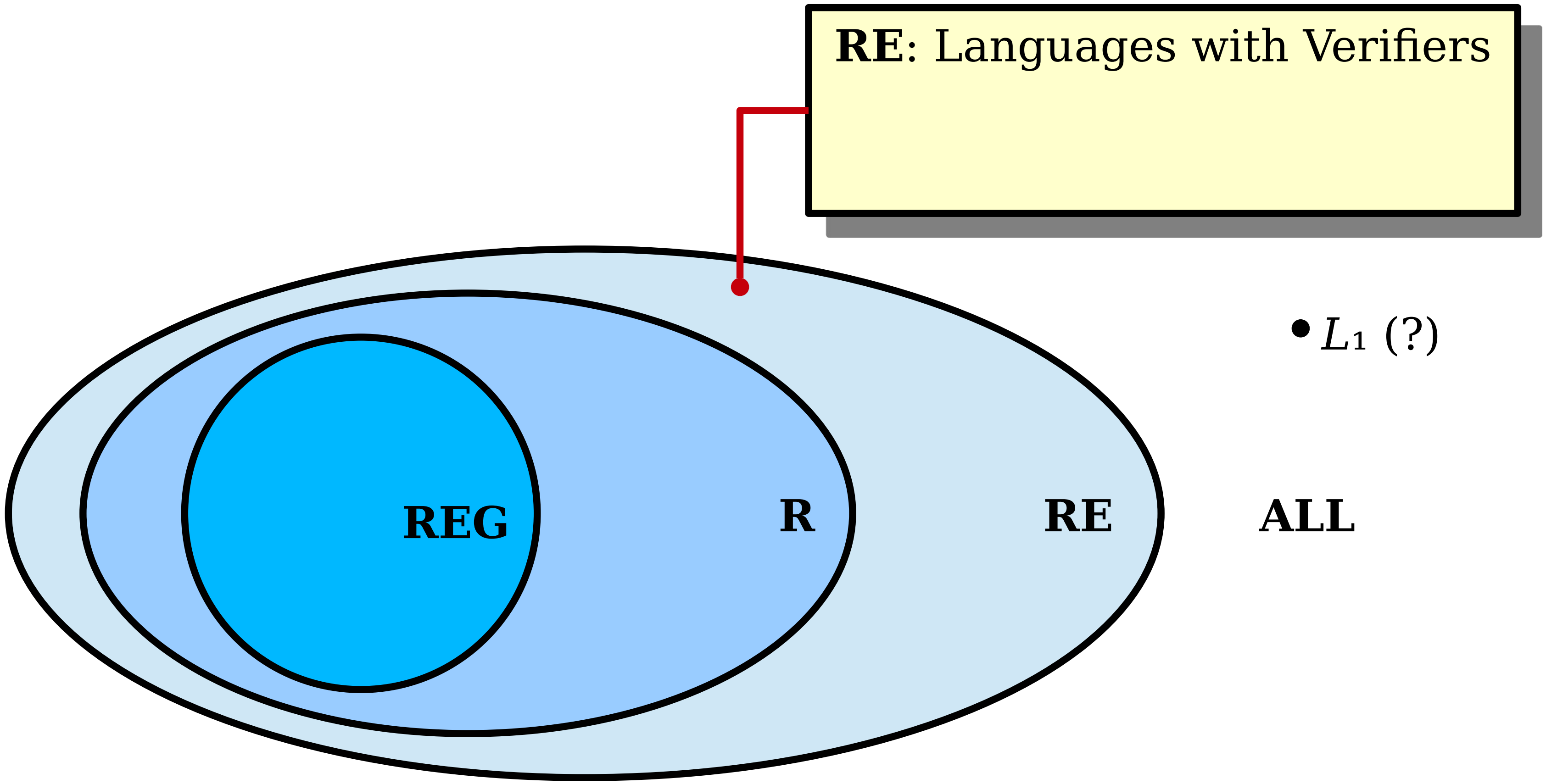
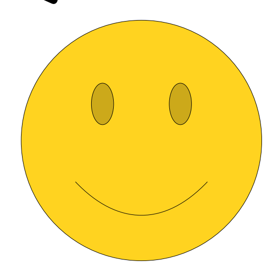


Specifically, we saw that **RE** is the class of languages that have verifiers.



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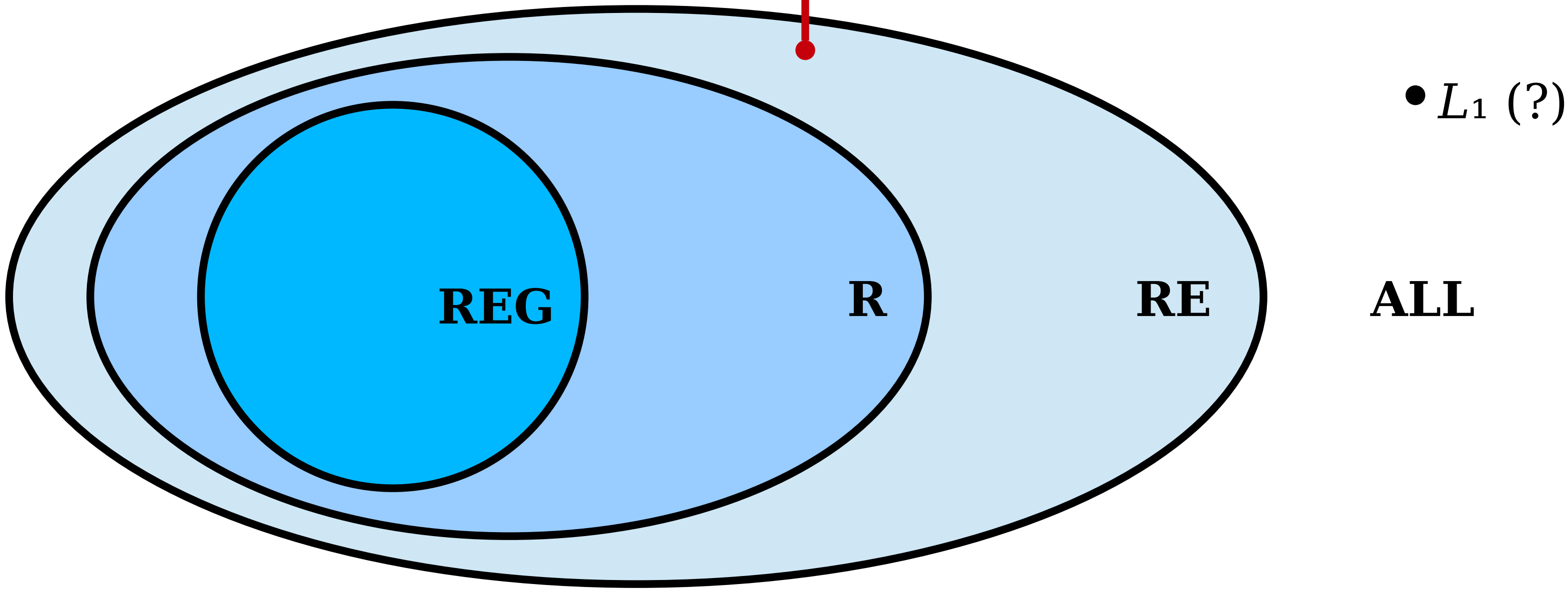
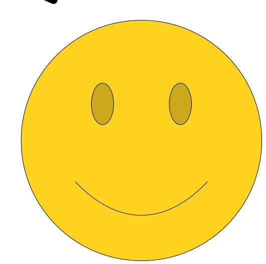
RE: Languages with Verifiers



If you think back to what a verifier for a language is supposed to do, at a high level, it's really an "answer checker."



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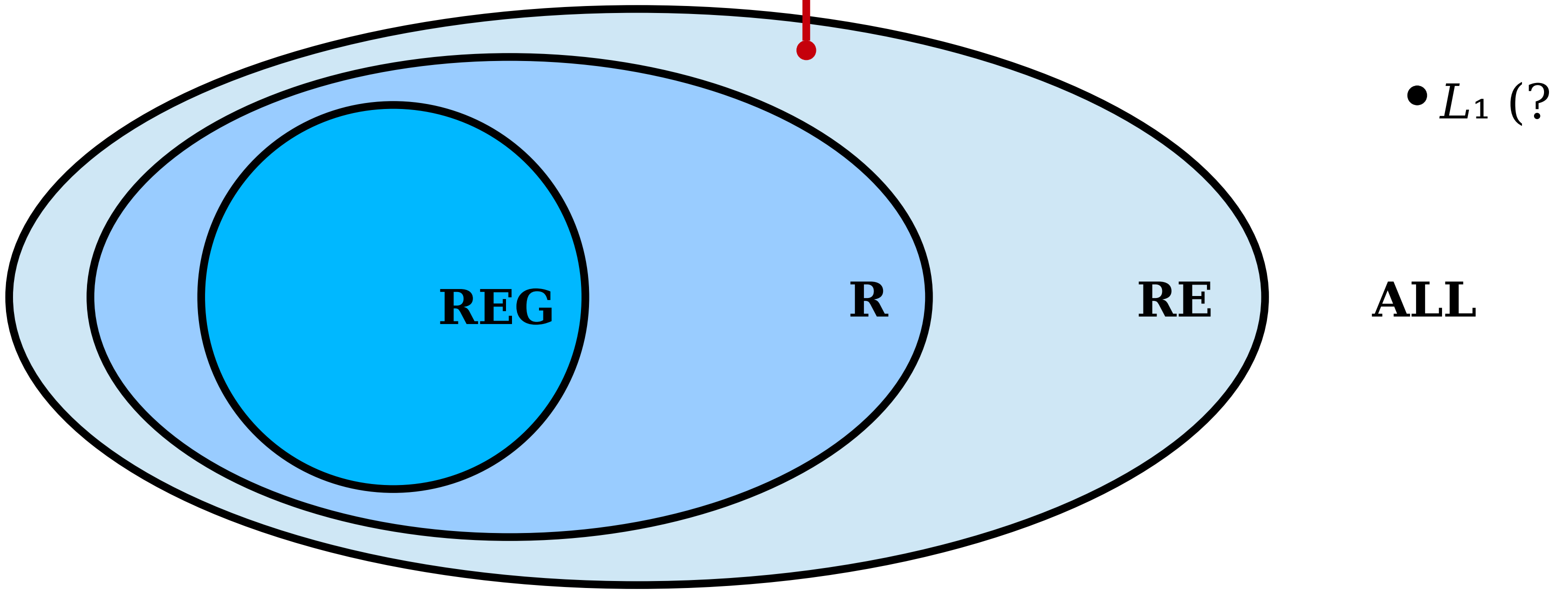
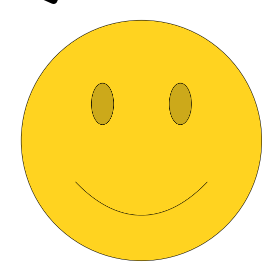


Specifically, a verifier is supposed to take in a string and a certificate, then see whether the certificate proves whether the string is in the language.



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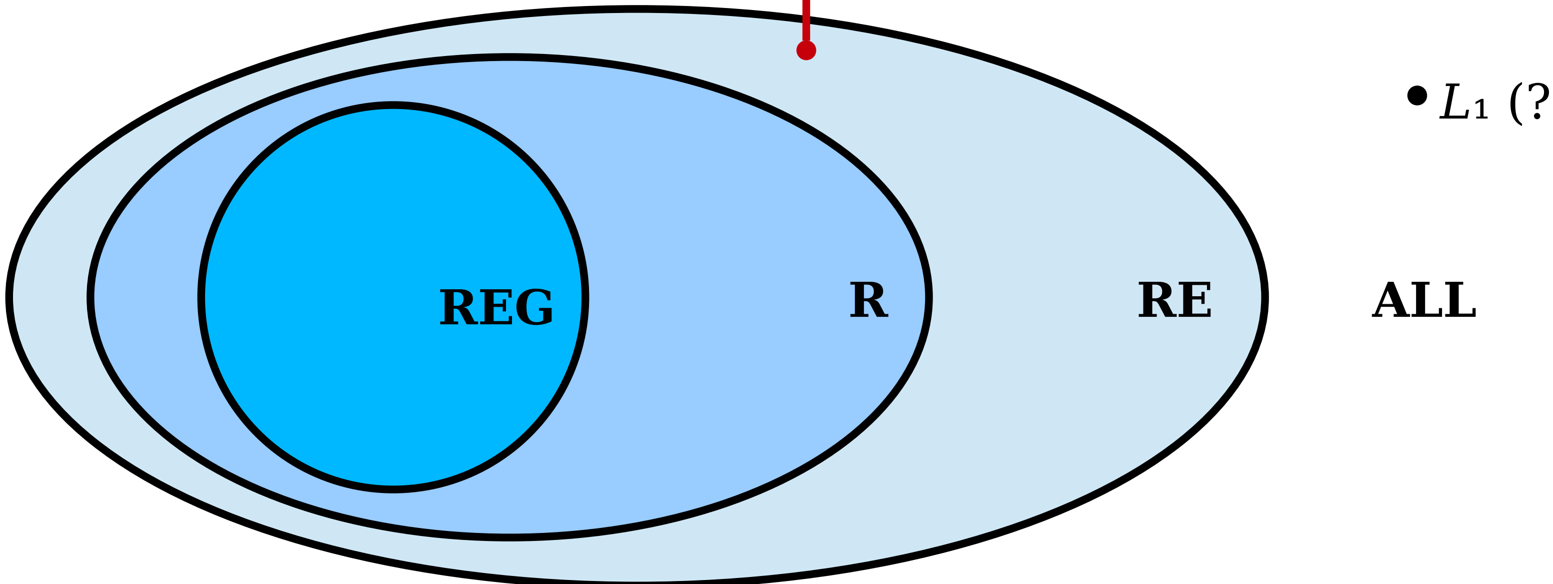
RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



In that sense, you can think of the **RE** languages this way: they're the languages where, for any string in the language, there's some way to <u>prove</u> that the string is indeed in the language.

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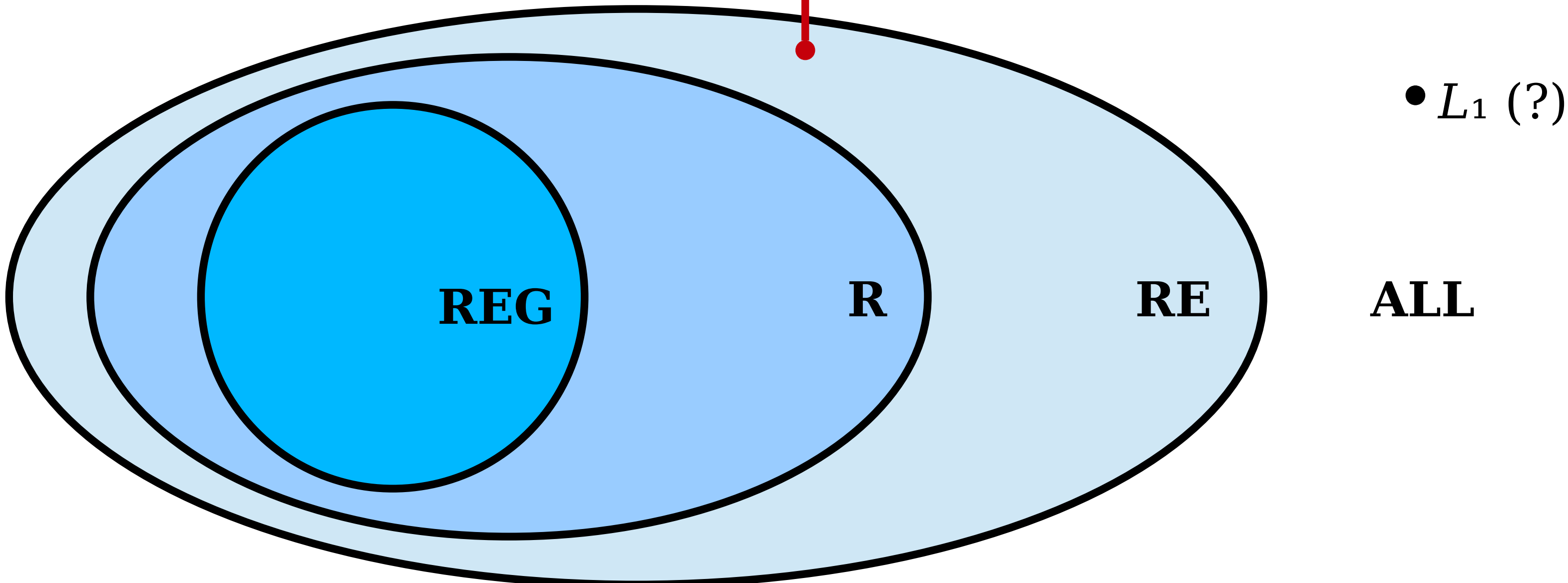

Turns out, this provides an amazingly good intuition for the **RE** languages. A language is in **RE** if and only if, whenever you have a string in the language, there's some way to prove it's in the language.

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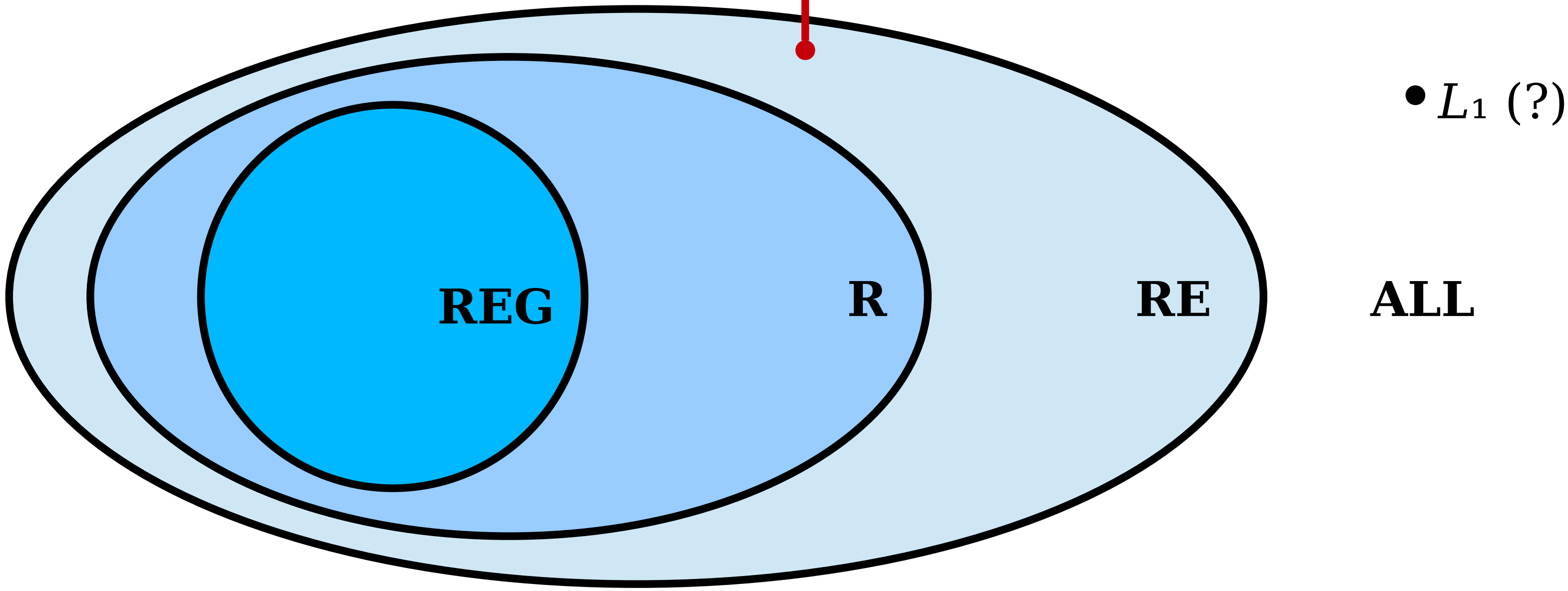

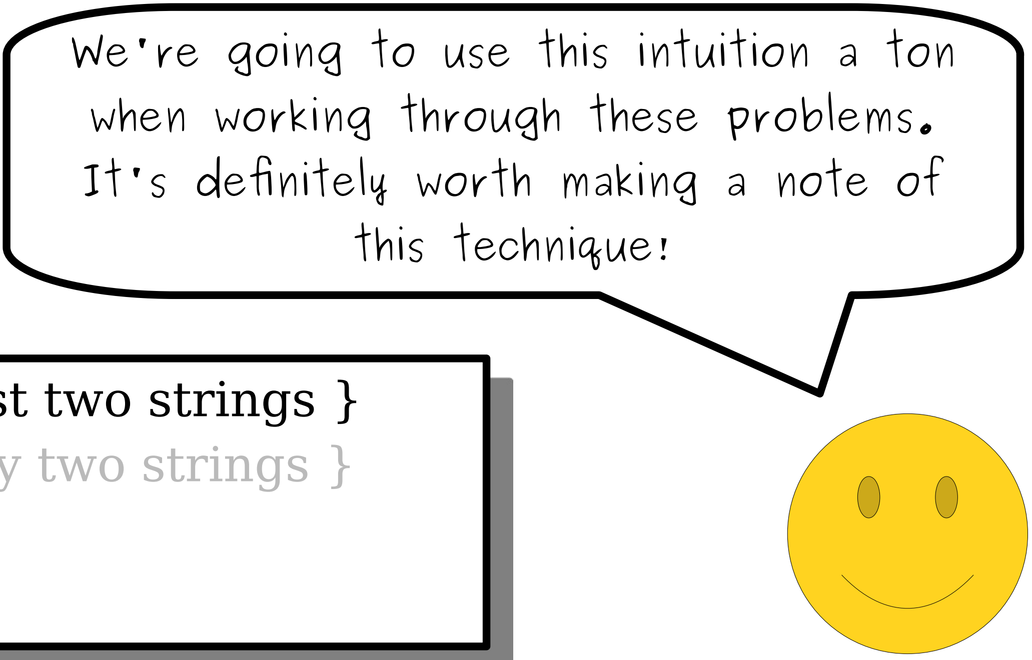

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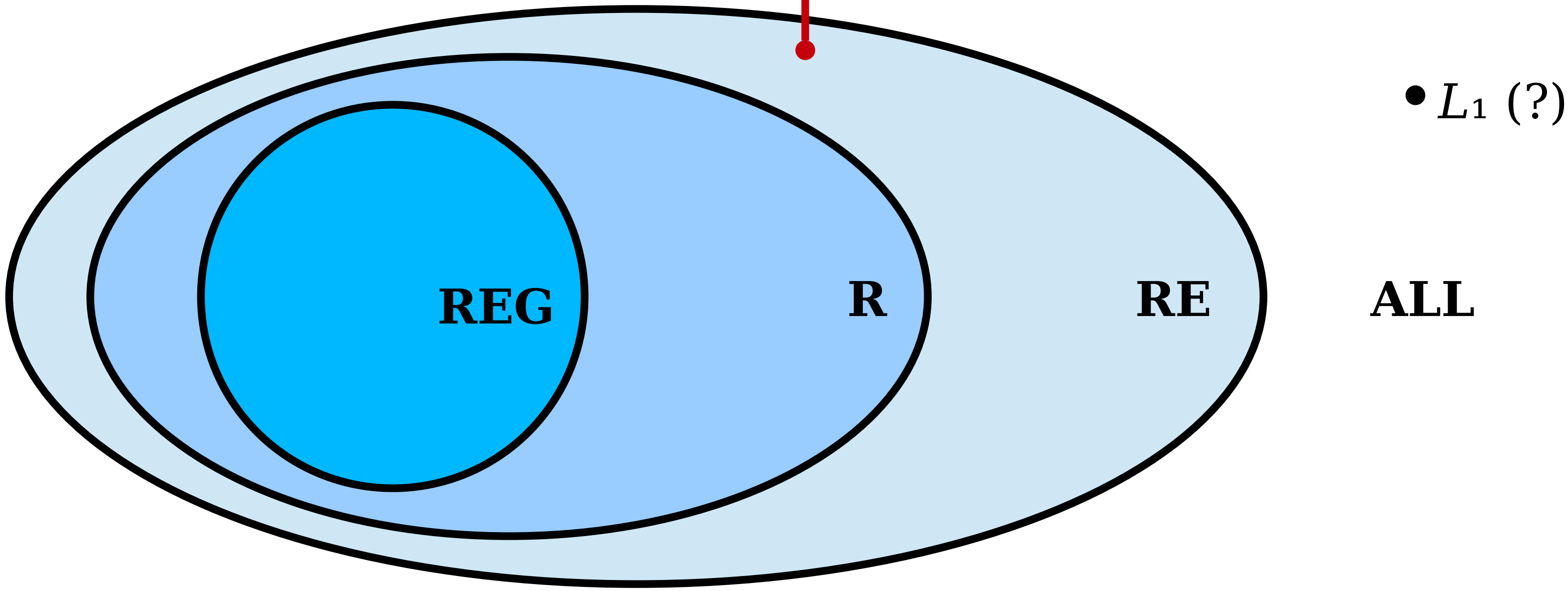

So let's go focus our attention to the particular language L_1 we have right now.

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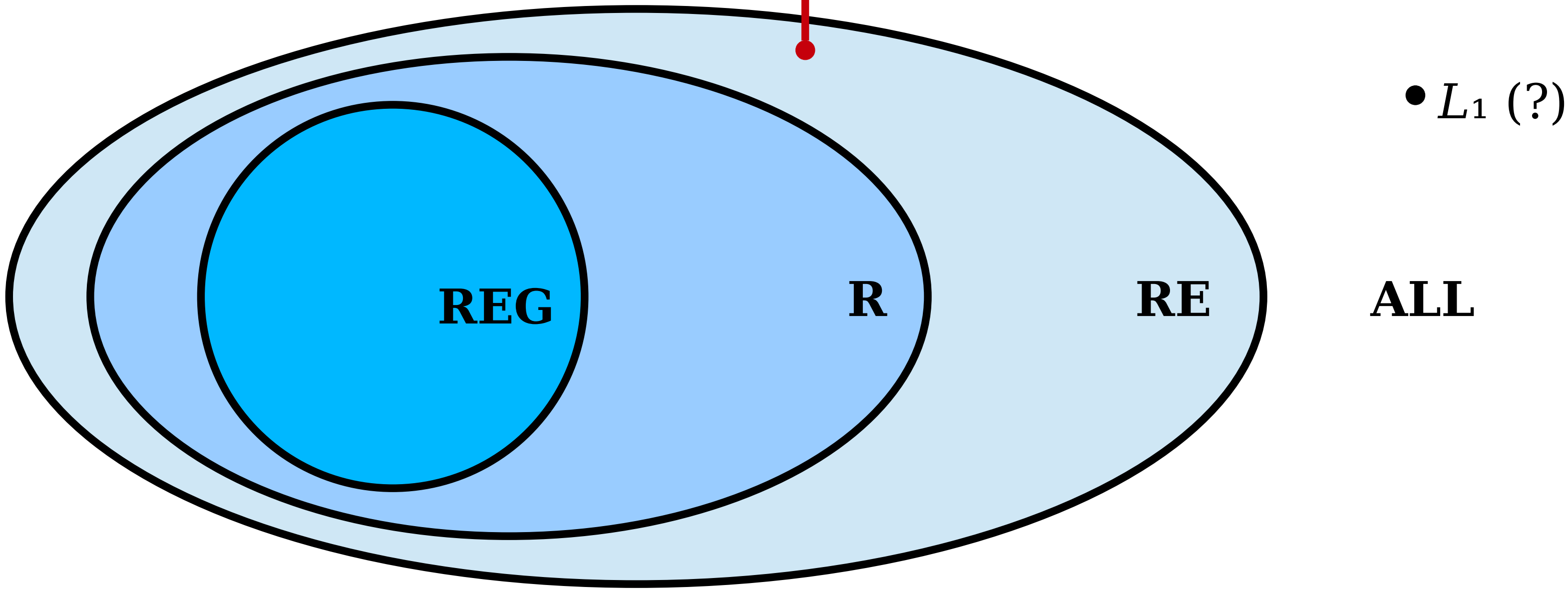

Imagine you have a string in L_1. What does that string look like?

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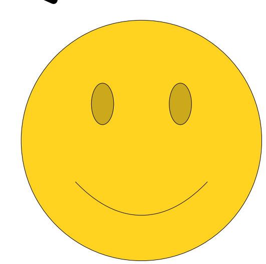

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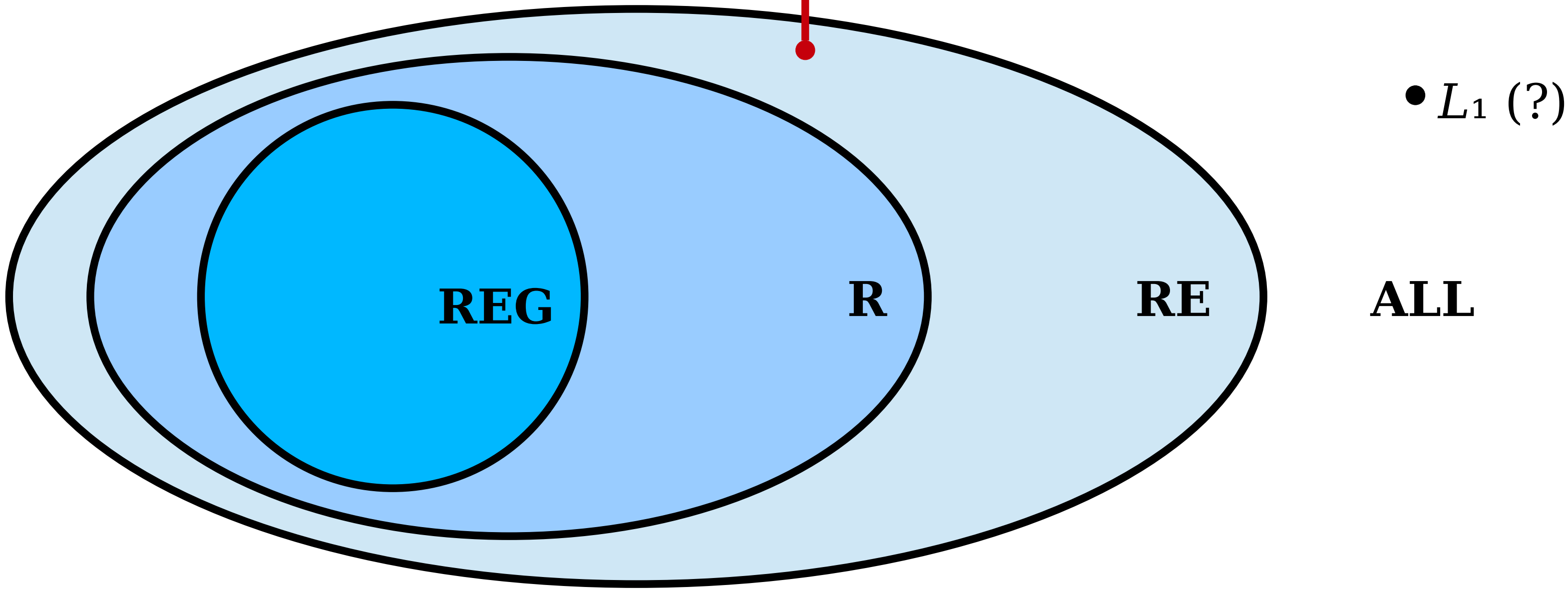
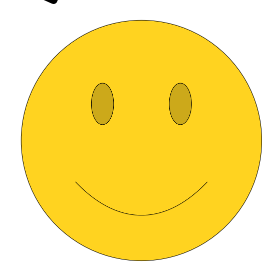


Well, according to the definition of the language, any string in L_1 must encode a TM that accepts at least two strings.



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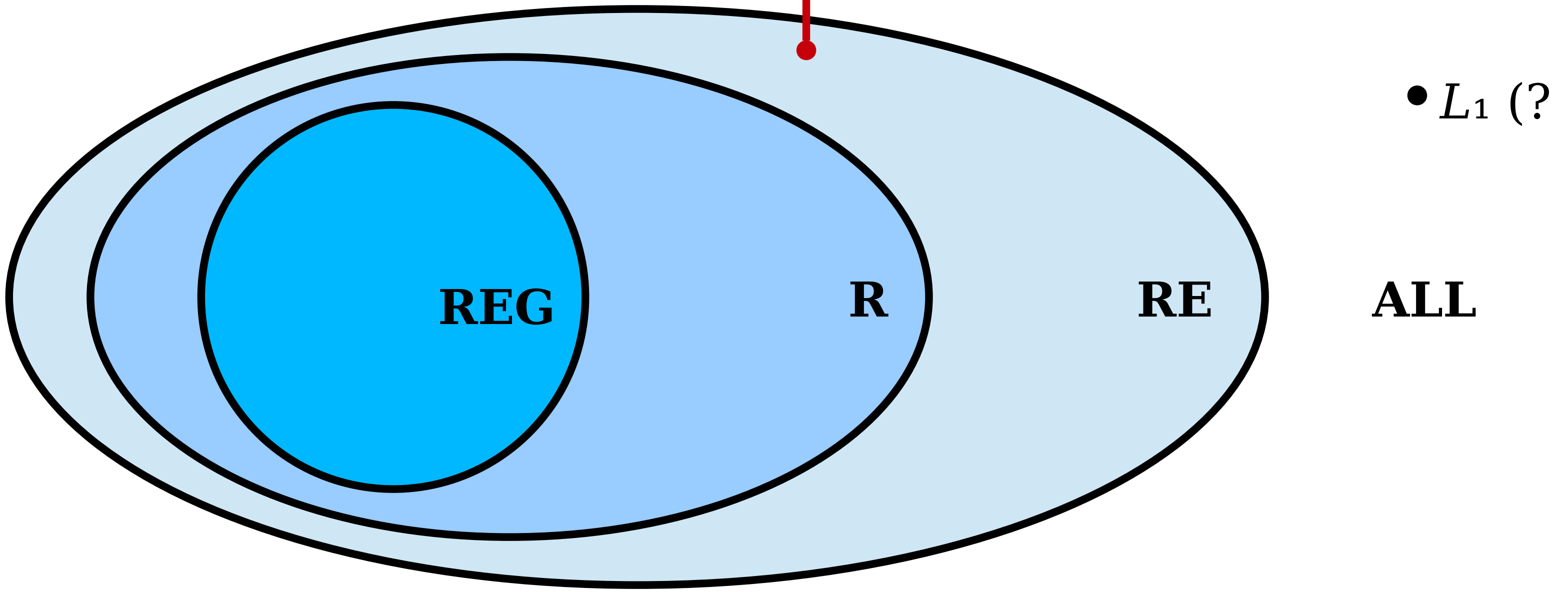
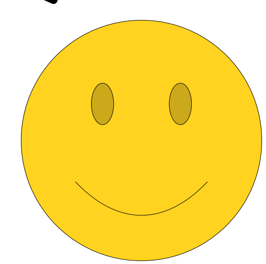


We can think of L_1 as "the language of TMs that accept at least two strings."



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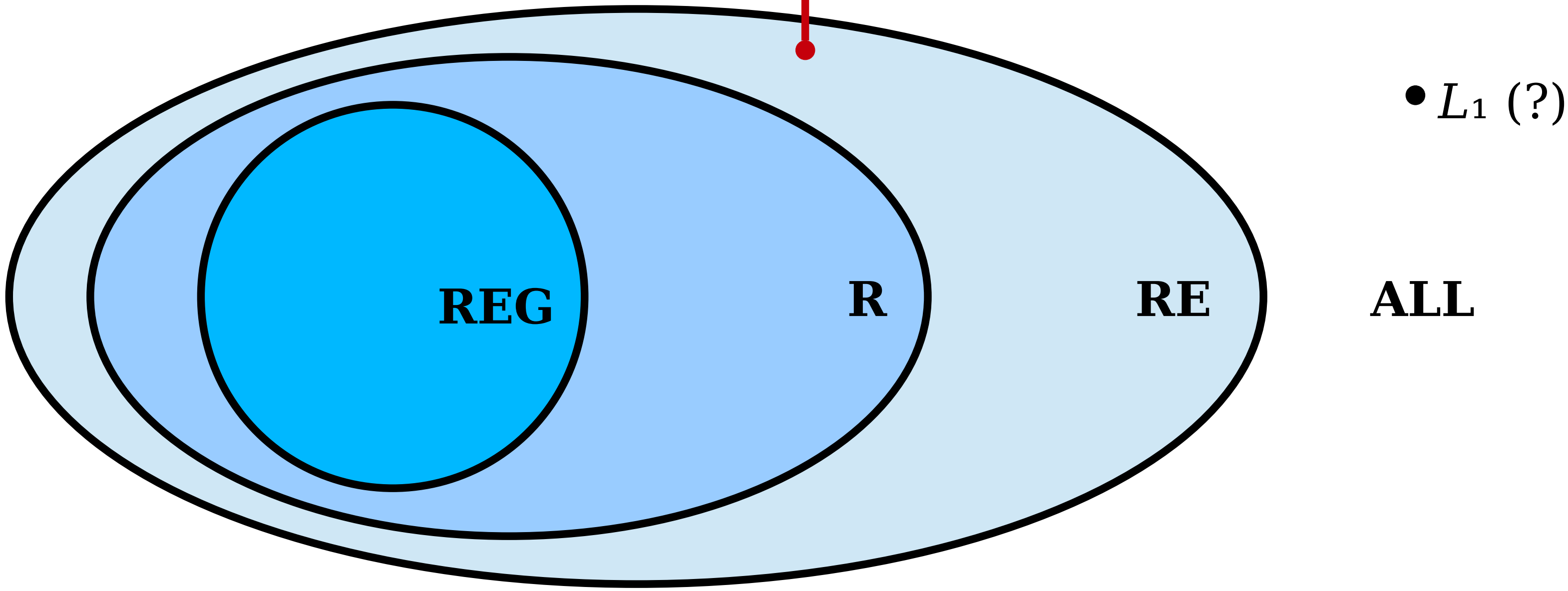

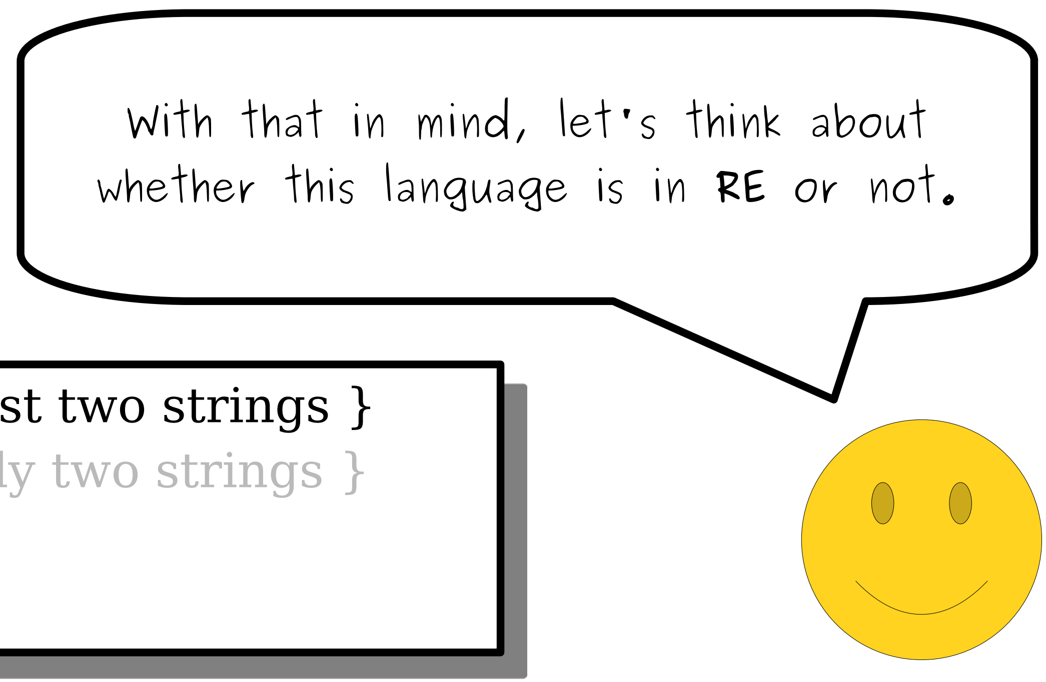

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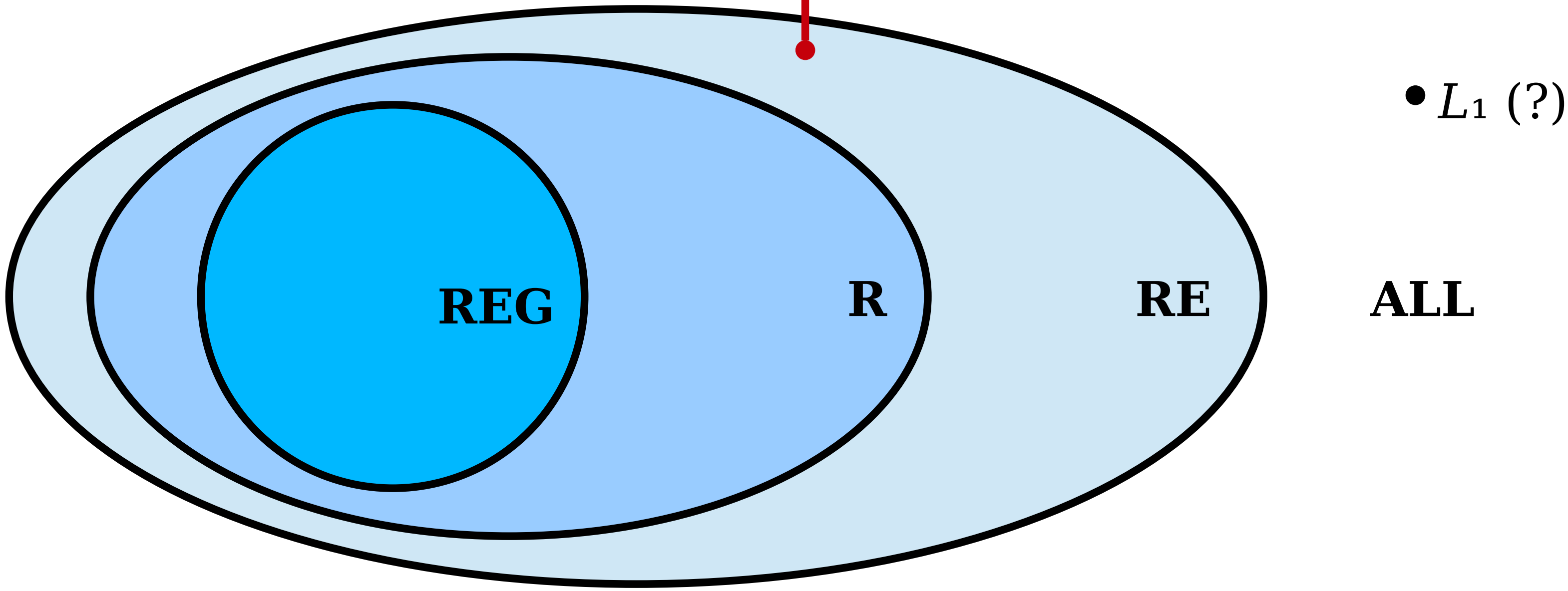
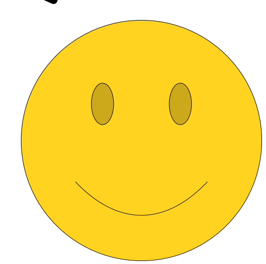


Let's imagine that we have a random TM and we are <u>*convinced*</u> that it accepts at least two strings.



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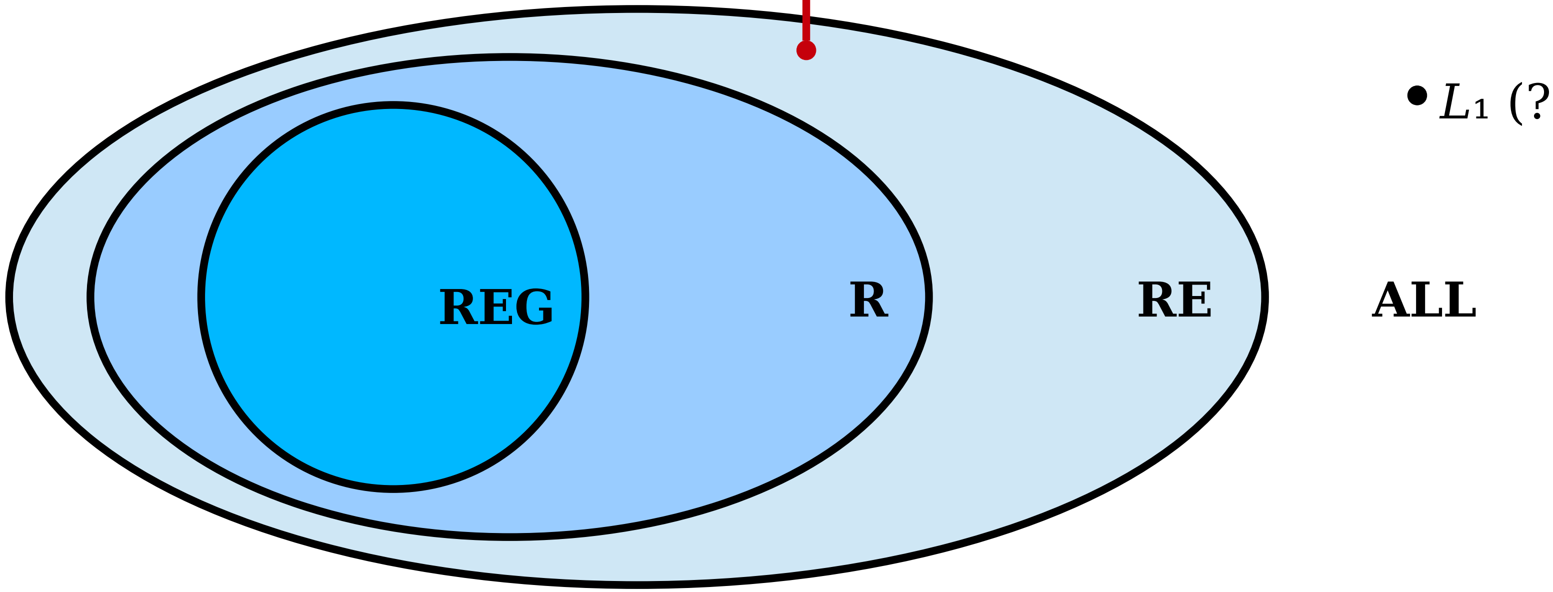
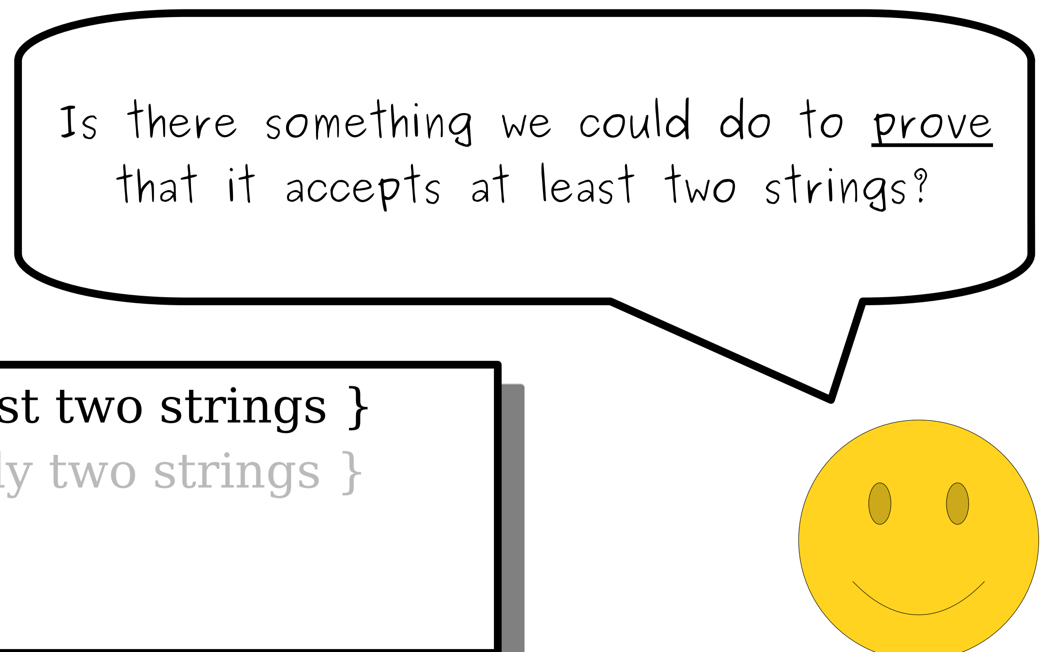
RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?





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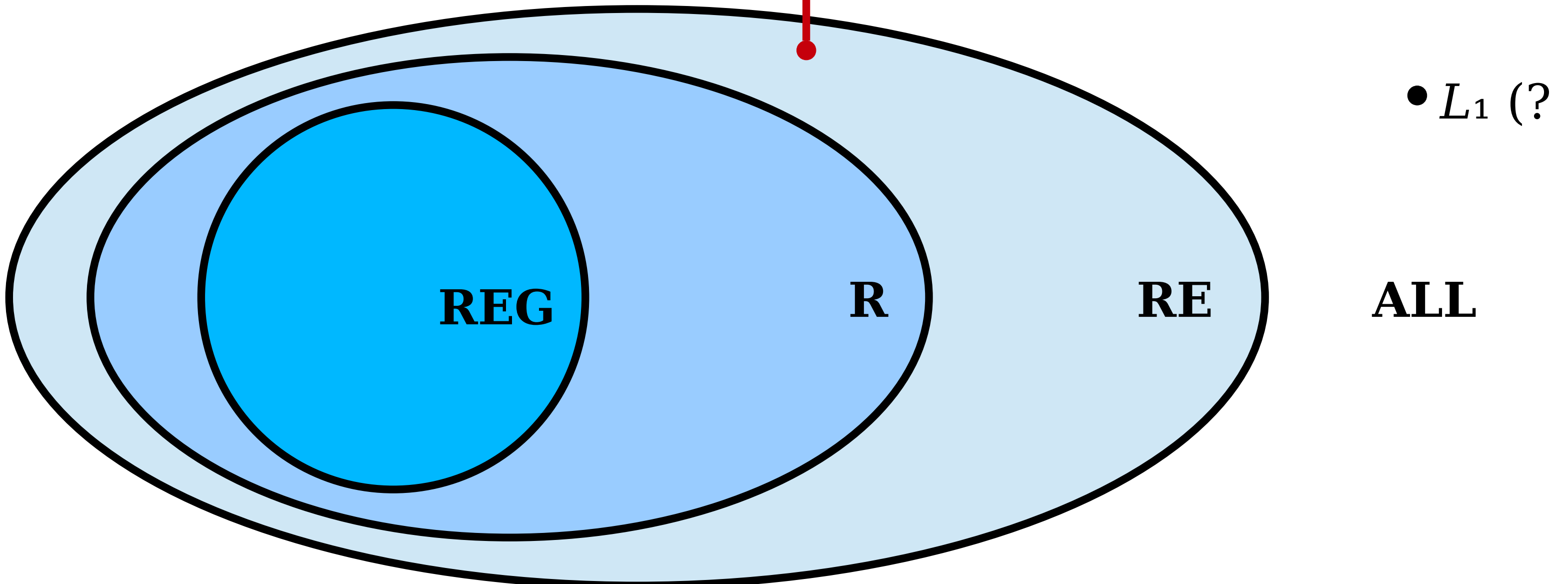

In other words, if we came across someone who was skeptical that the machine actually accepts at least two strings, could we convince them that the machine indeed does accept at least two strings?

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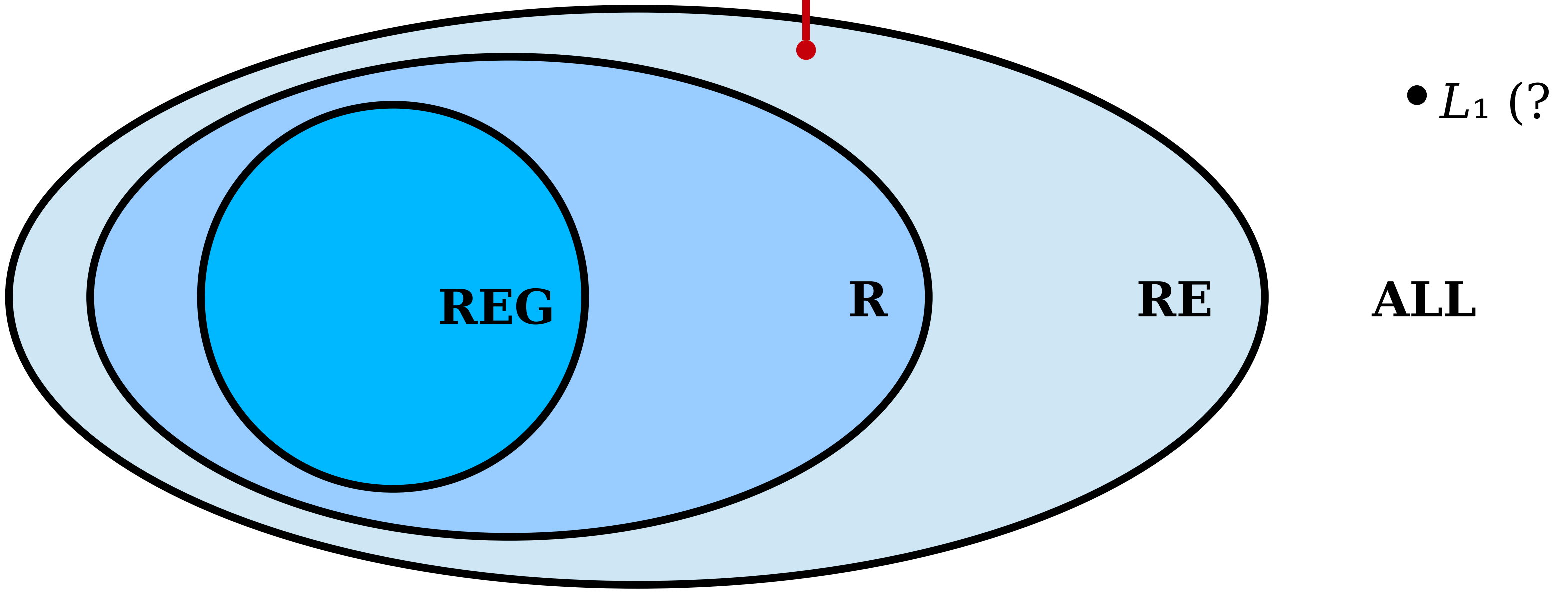

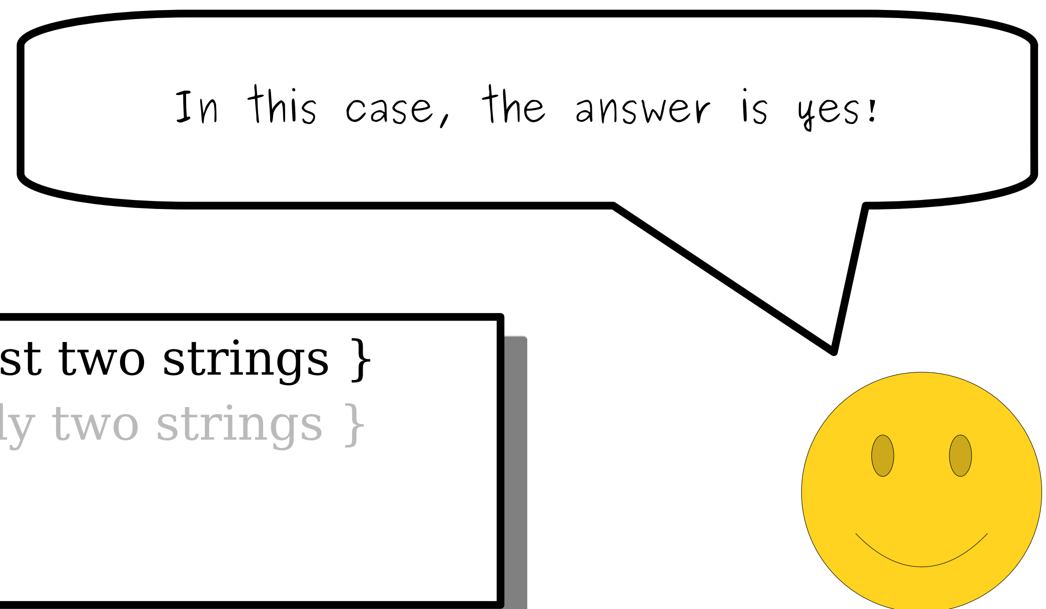

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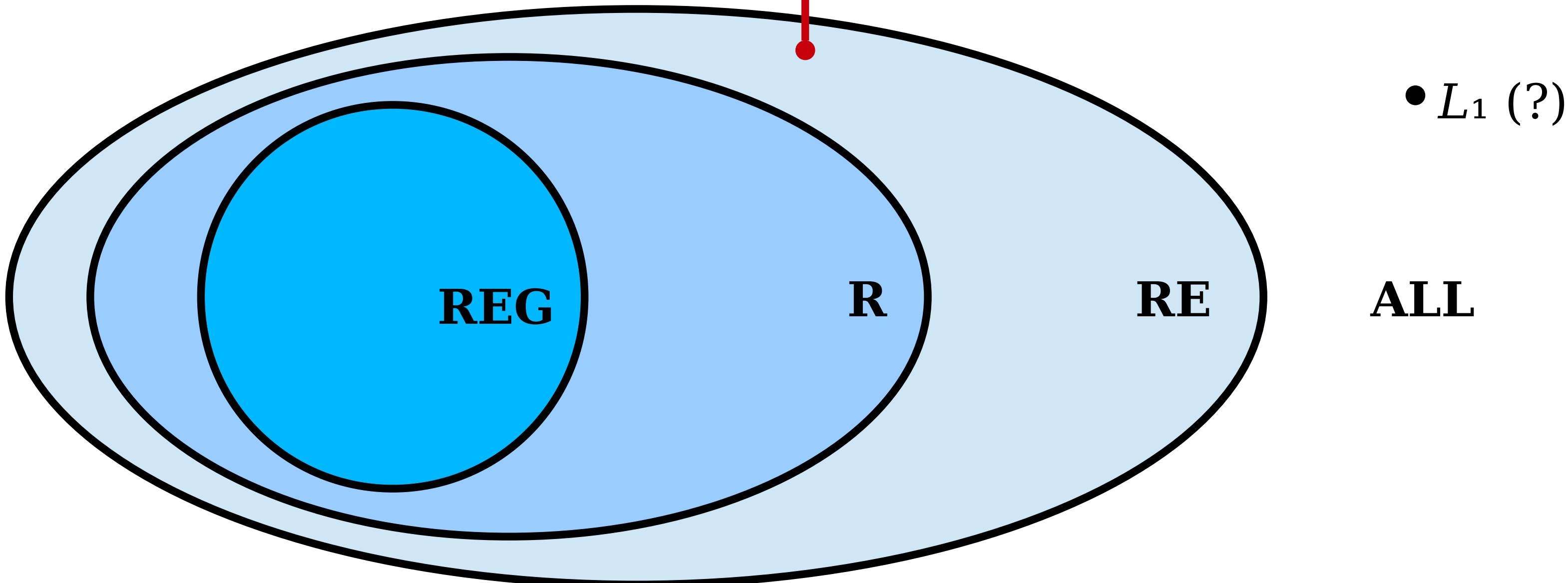

If we happened to know at least two strings that the machine accepted, we could just run the machine on both those strings and watch it accept them.

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RE: Languages with Verifiers

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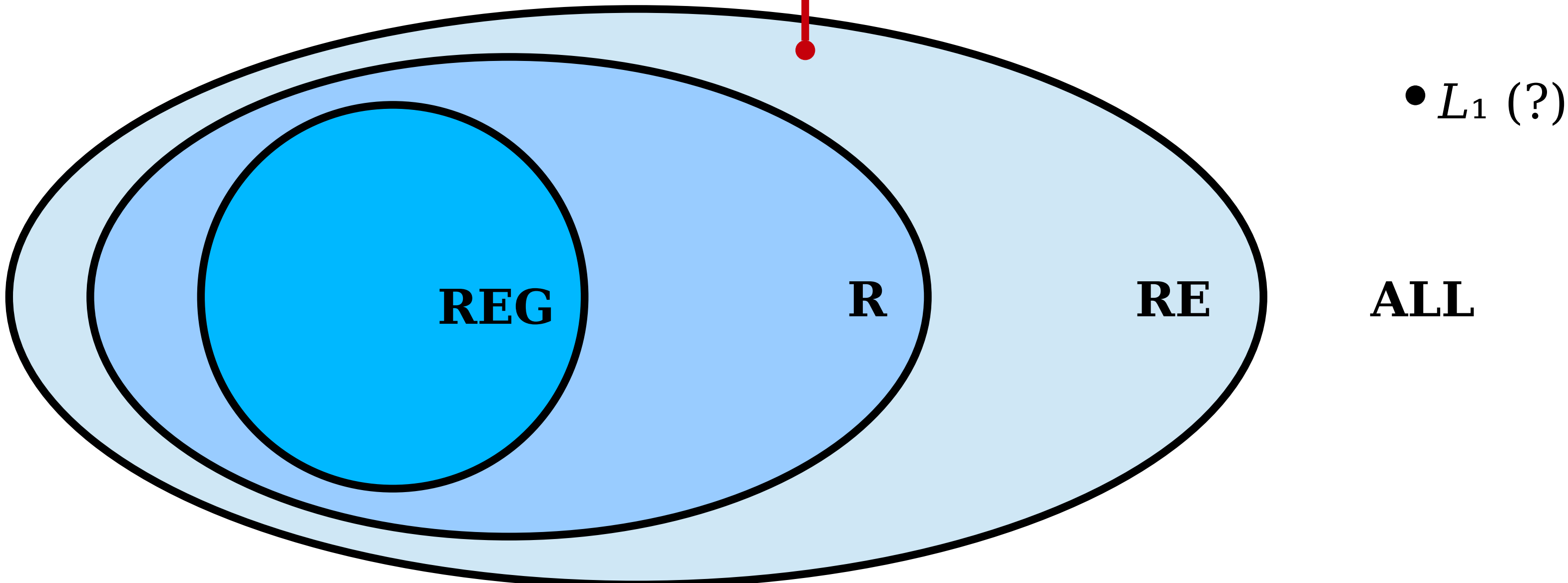

Anyone who was initially skeptical that our TM accepted at least two strings would definitely be convinced at that point. They just watched the TM accept at least two strings!

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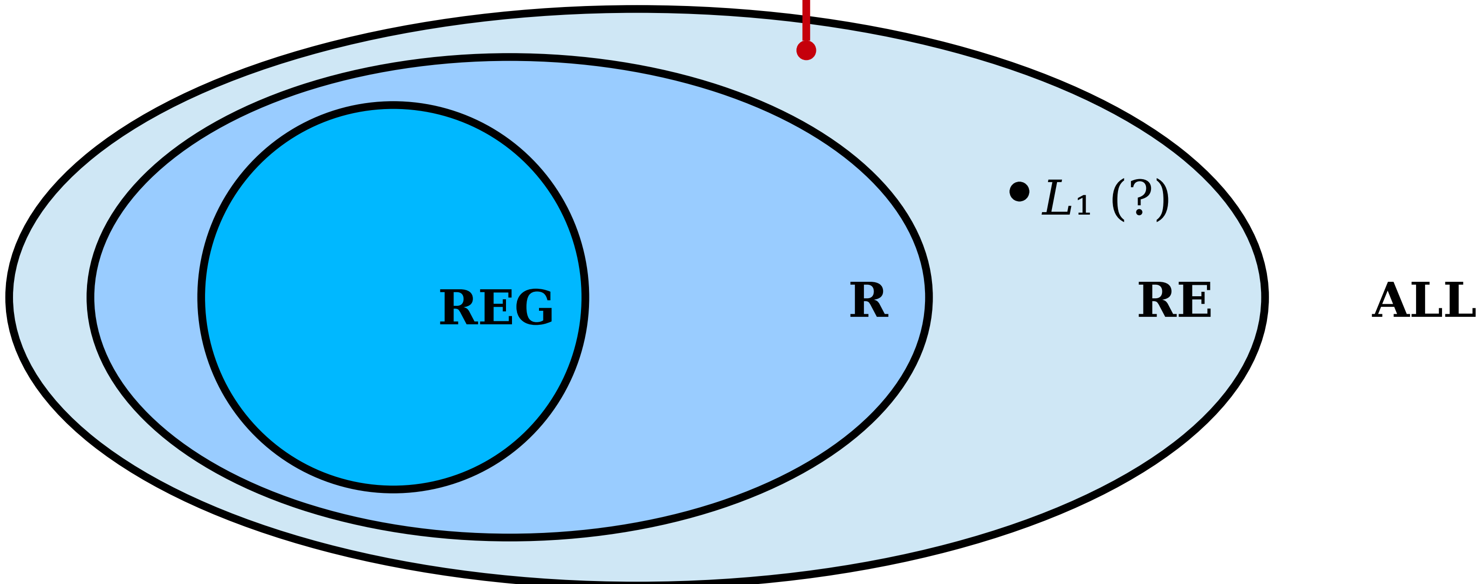
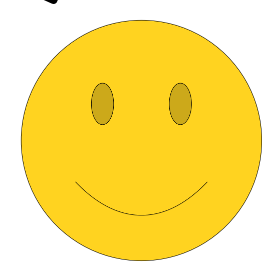


So, going off this intuition, we can be reasonably confident that the language L_1 is indeed in **RE**.



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RE: Languages with Verifiers

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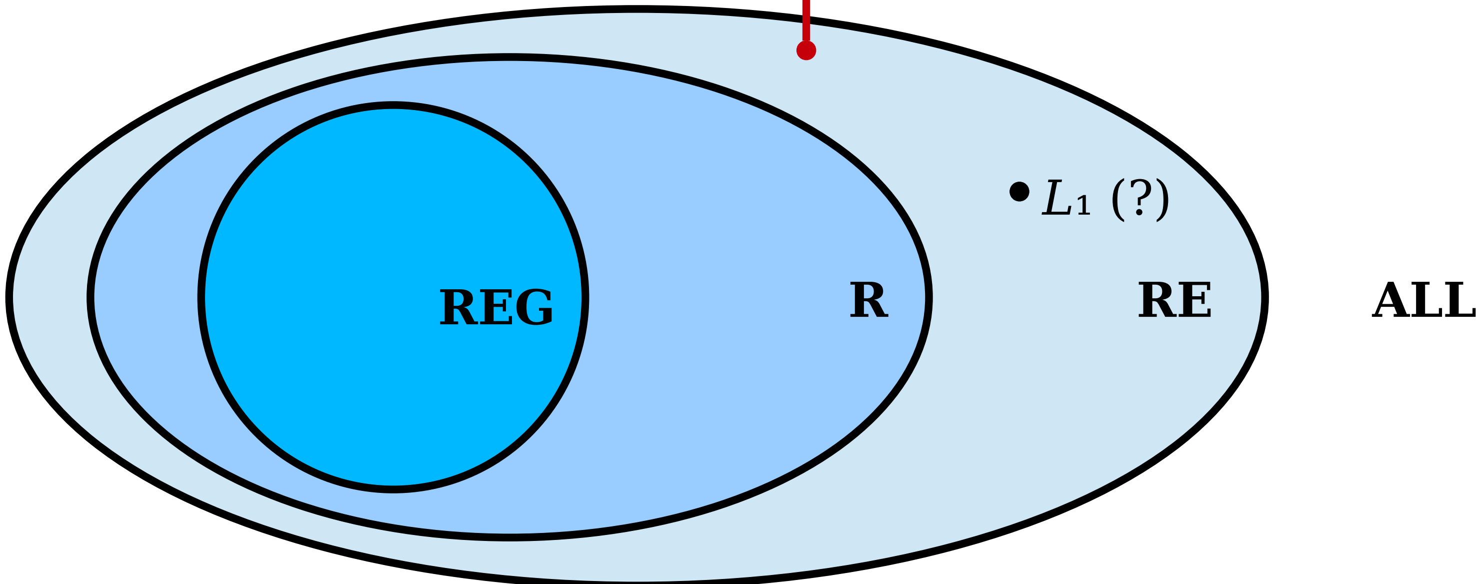
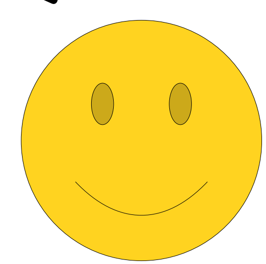


At this point we haven't ruled out the possibility that it's also in **R** or is regular, but it's almost certainly not outside **RE**.



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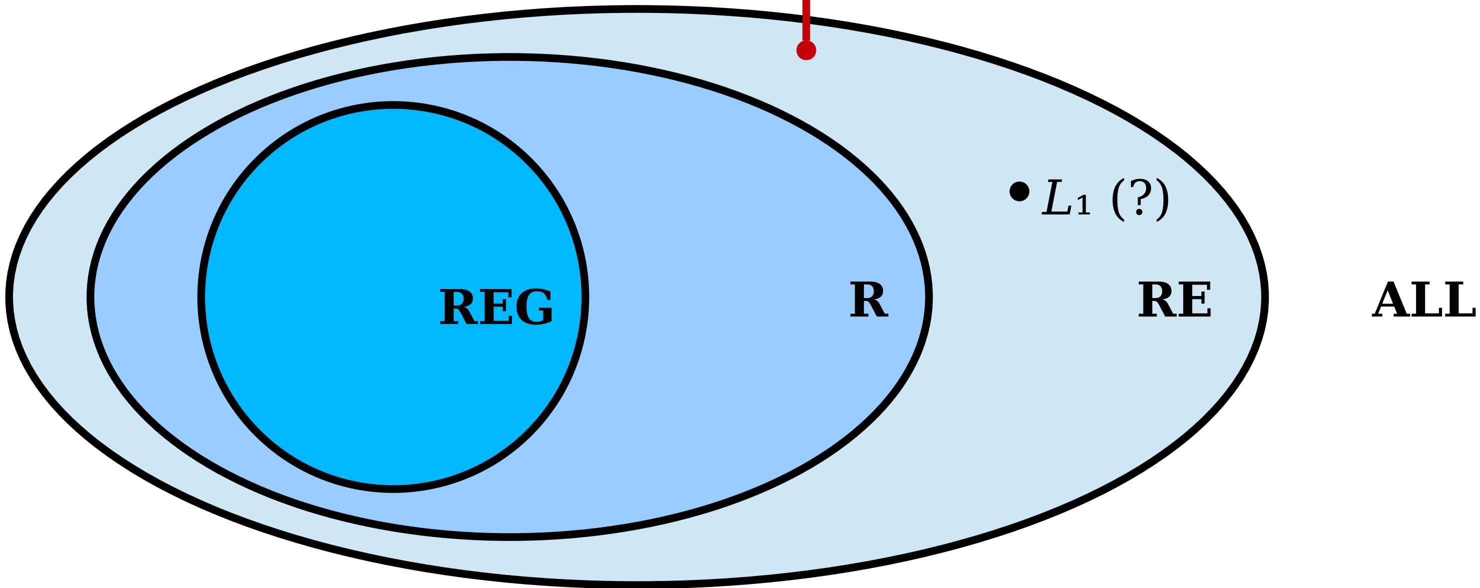
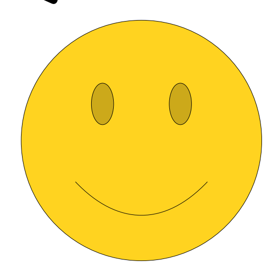


Although the question here was just to go and place L_1, it's not a bad idea to think about how we'd actually go and build a verifier for L_1.



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RE: Languages with Verifiers

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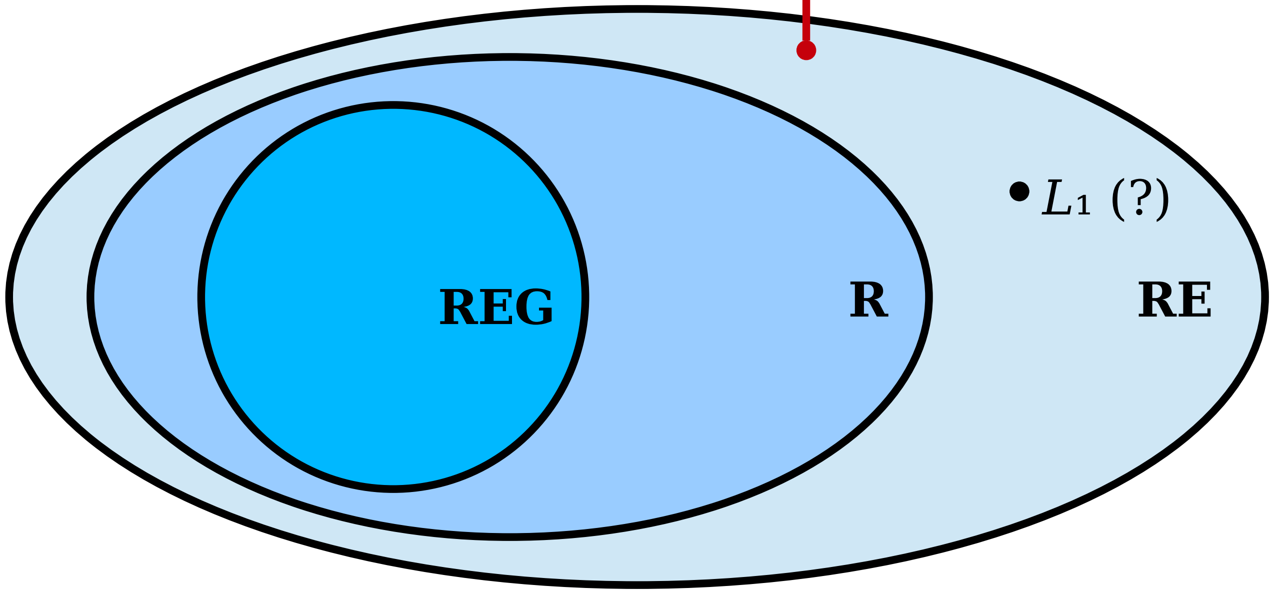
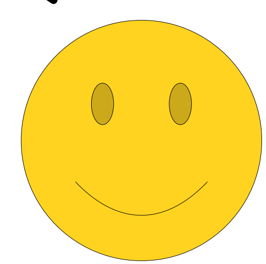


The idea would go something like this.



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RE: Languages with Verifiers

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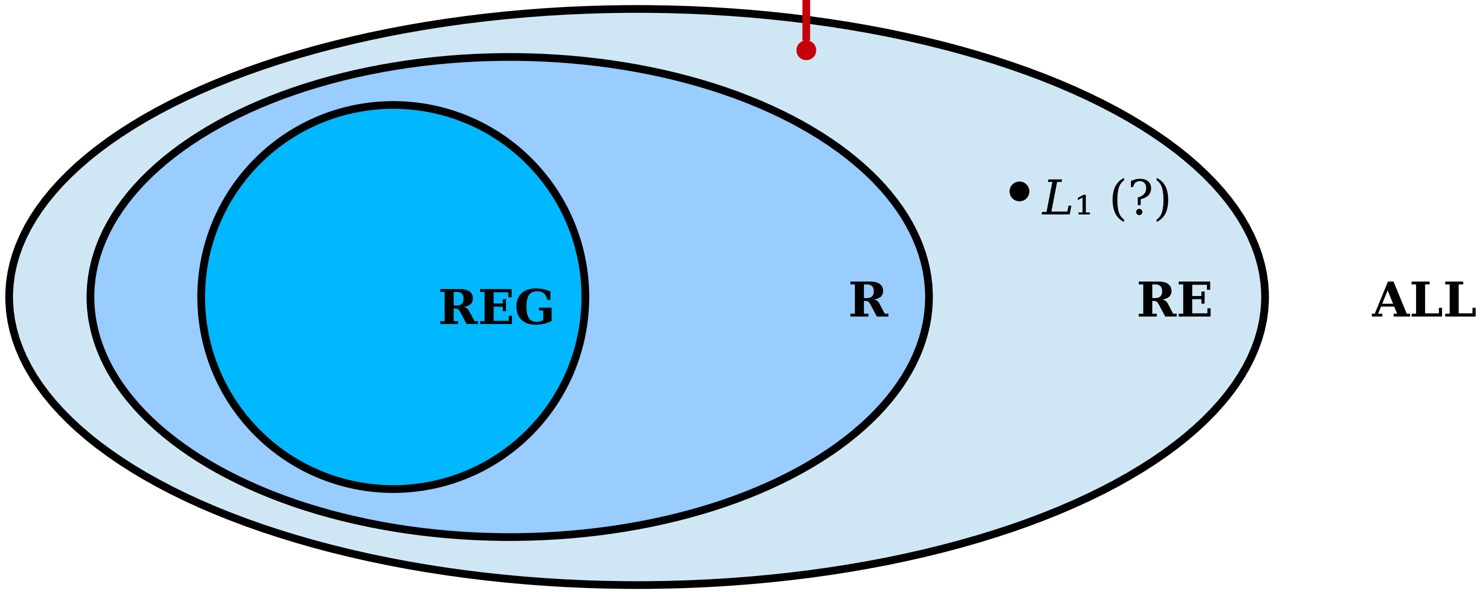
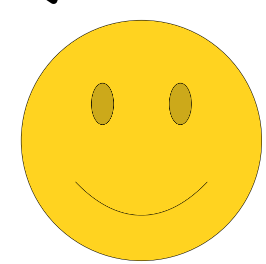


We can prove that our TM M accepts at least two strings by telling our verifier what two strings M is going to accept.



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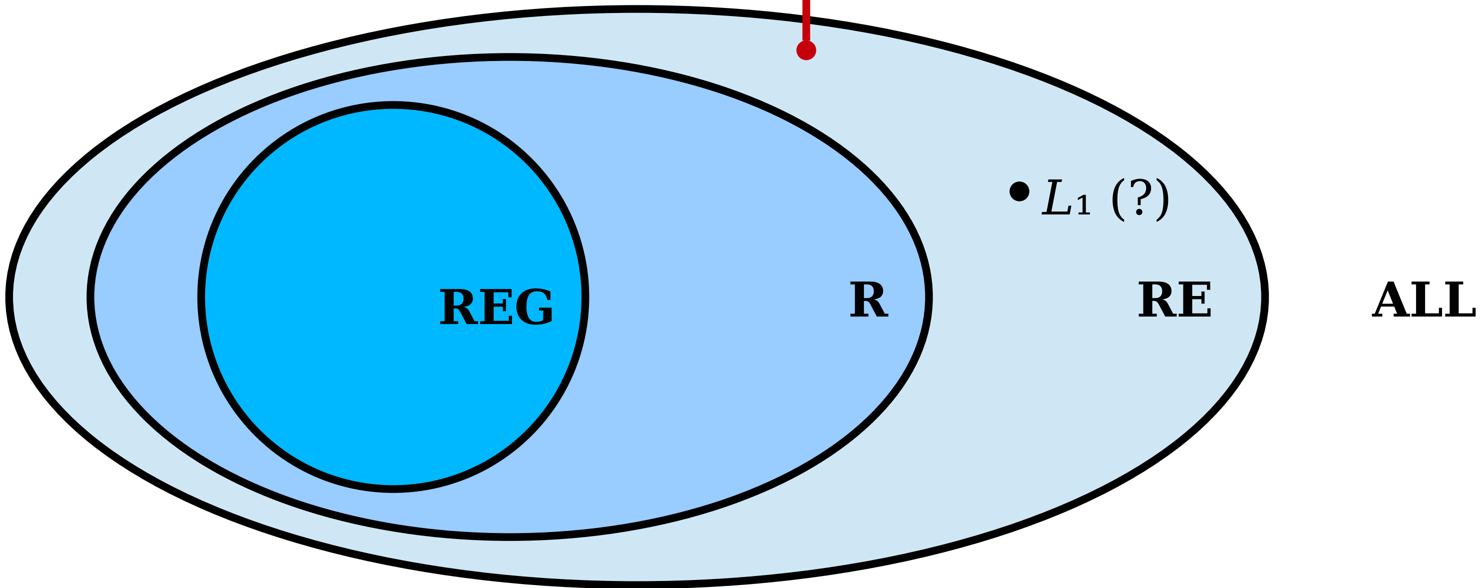

To ensure that our verifier doesn't go into an infinite loop (remember – verifiers aren't allowed to loop!), we can also give the verifier the number of steps it's going to take for M to accept.

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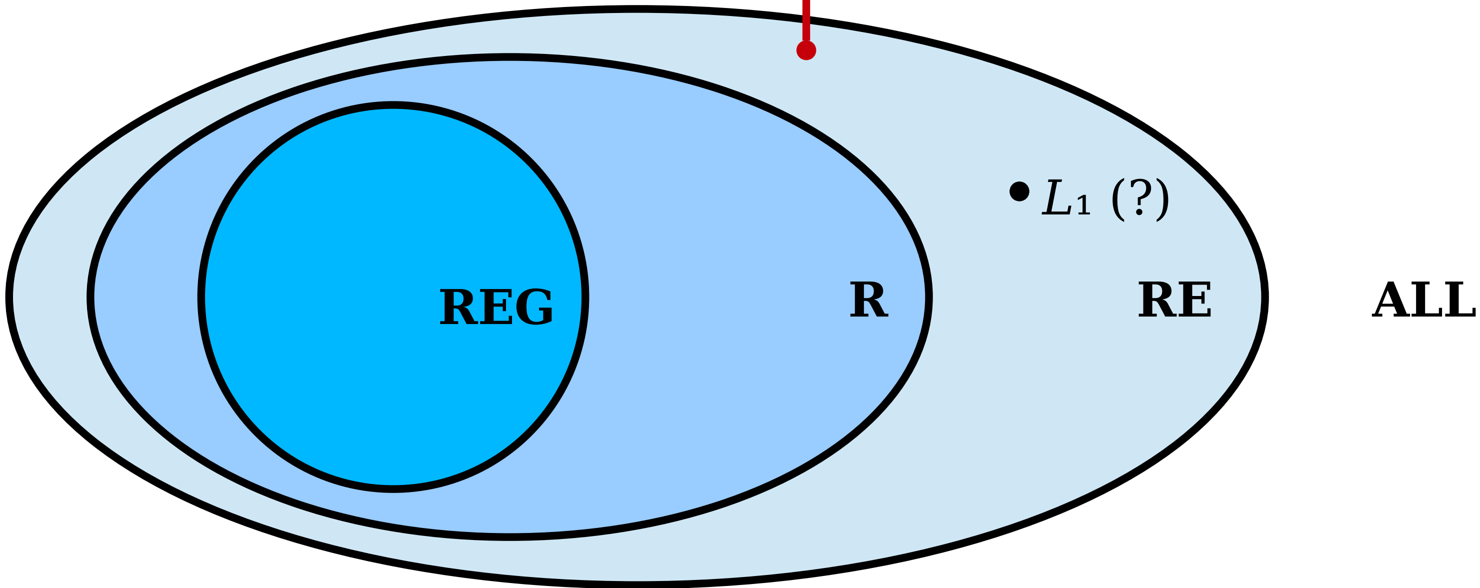

So the verifier would take in as input the TM M, two strings w_1 and w_2, and a number of steps n, and could run M on the stings w_1 and w_2 for up to n steps.

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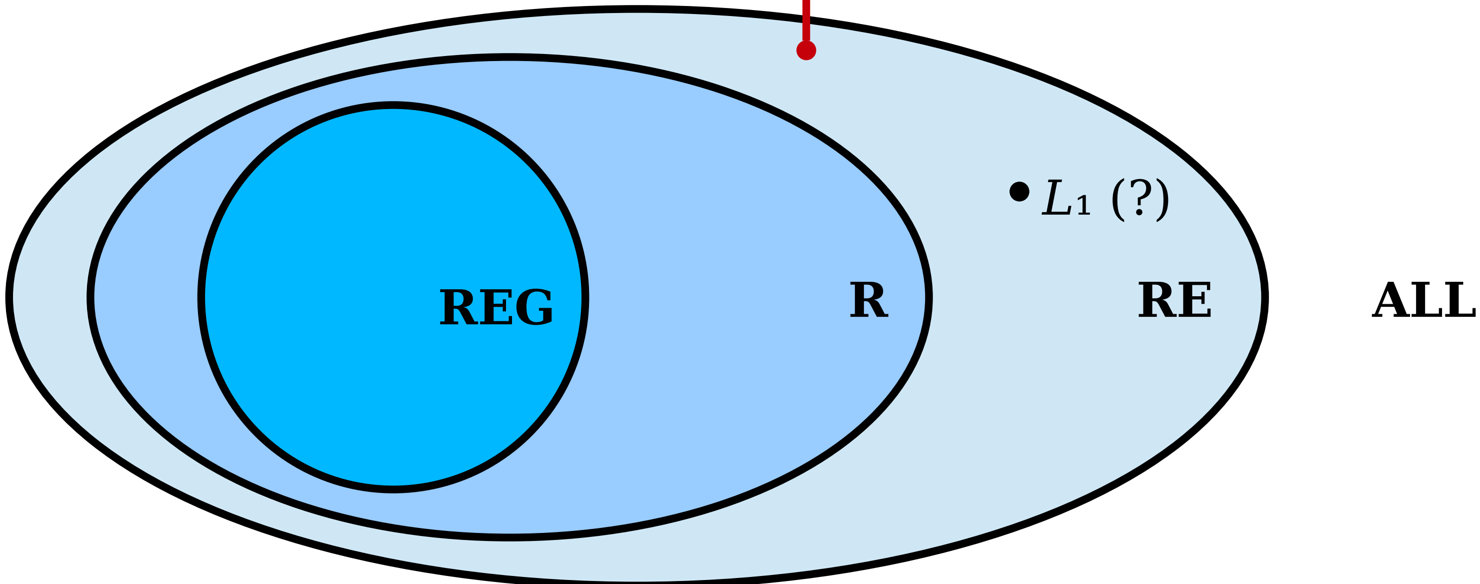

If M accepts both w_1 and w_2 within that many steps, then the verifier is convinced that M definitely accepts at least two strings.

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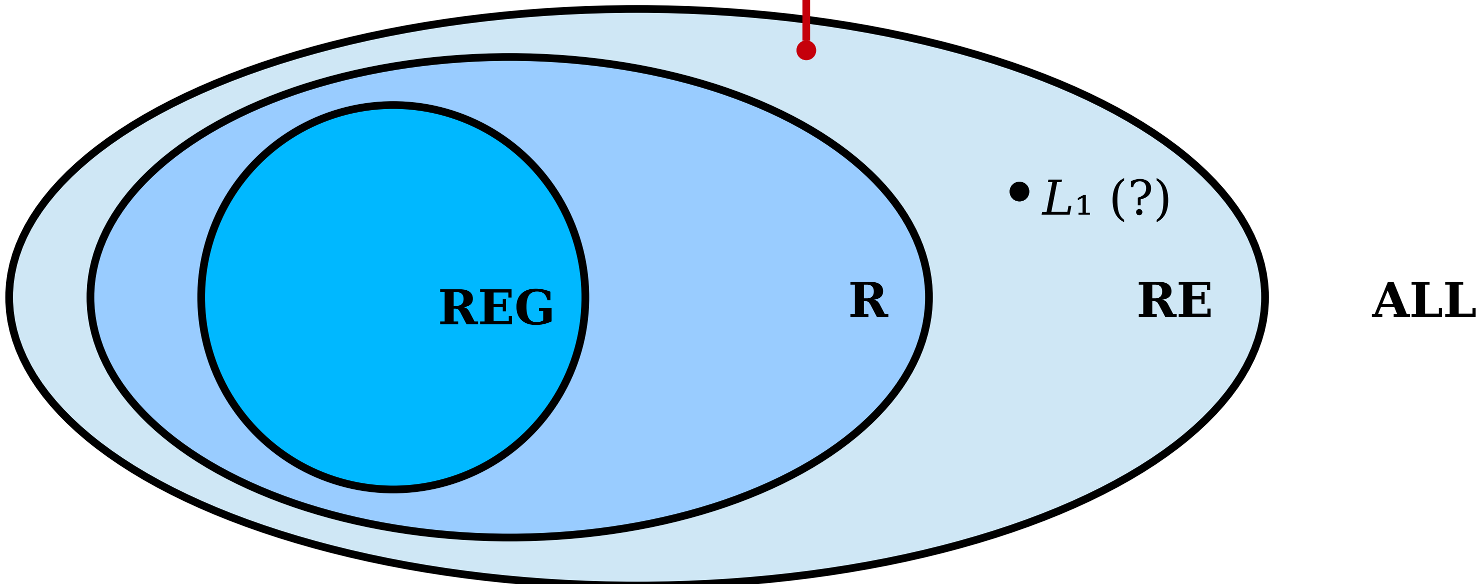
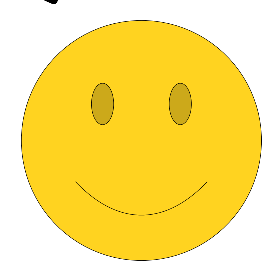


If that doesn't happen, the verifier isn't sure of what the answer is. Maybe M does accept two strings and we gave the verifier the wrong strings, or maybe we gave it the wrong number of steps.



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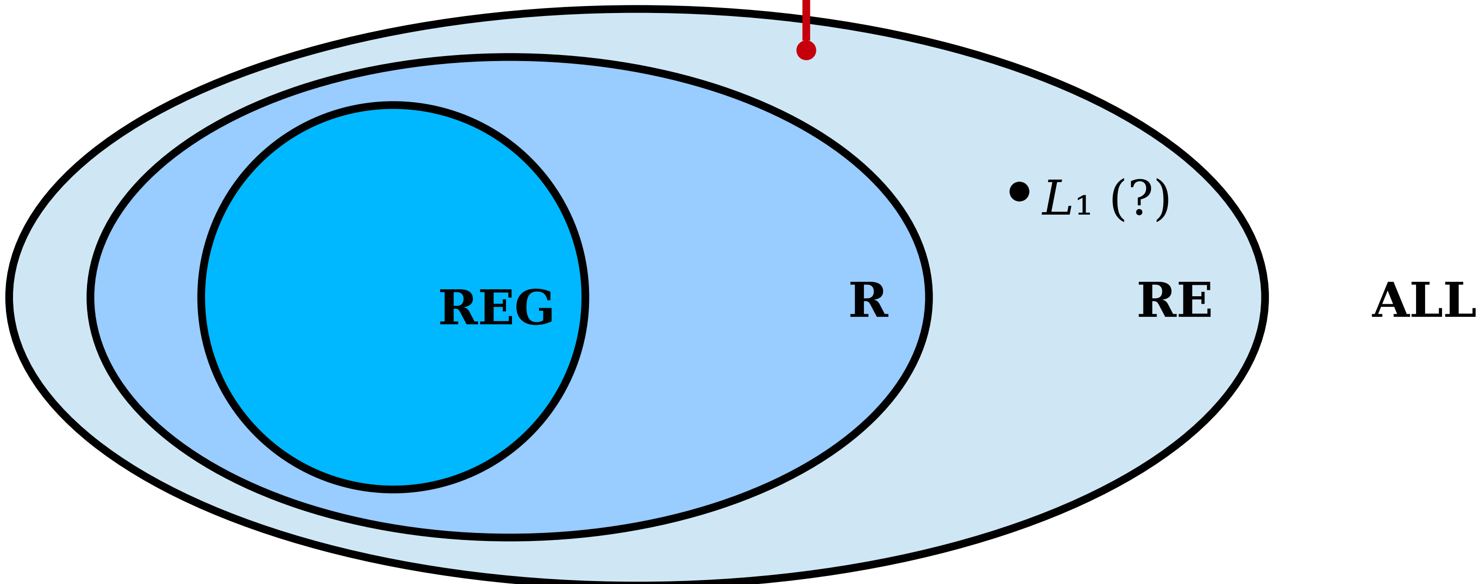

If you wanted to write this up as a formal proof, it's a good exercise! For now, though, we're just going to continue working through figuring out where this language goes on the Lava Diagram.

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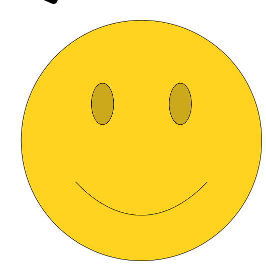

RE: Languages with Verifiers

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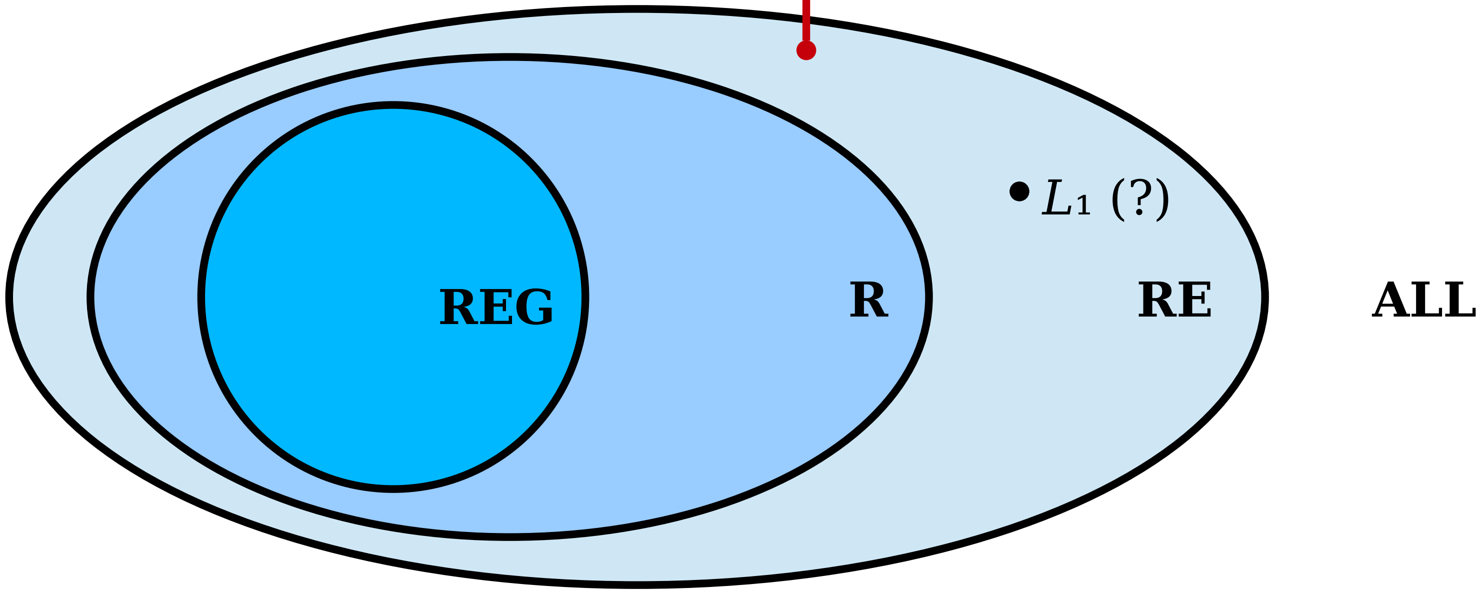

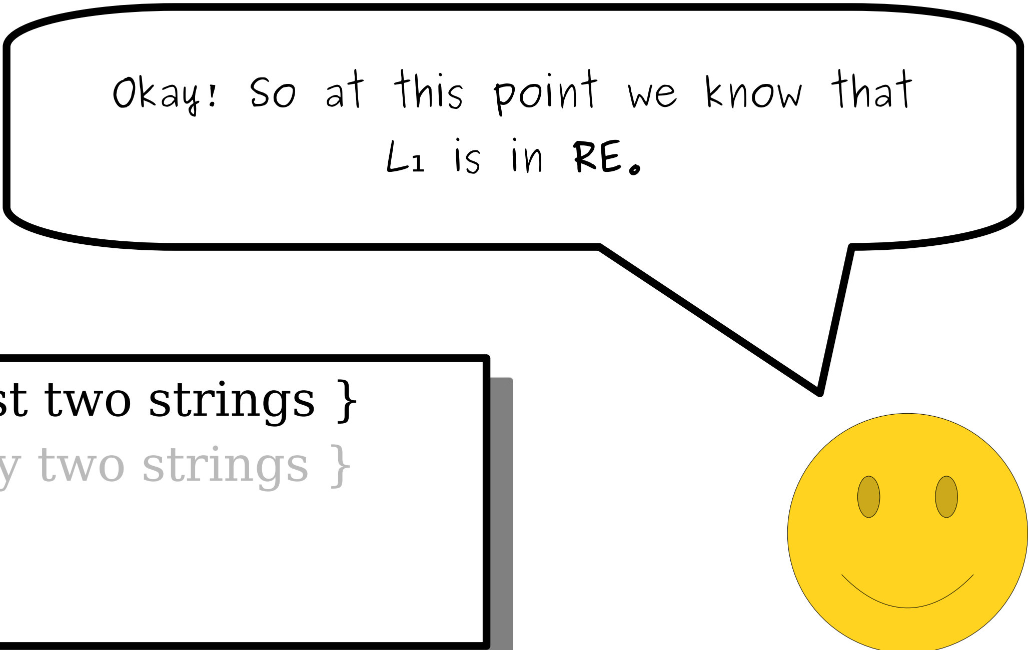

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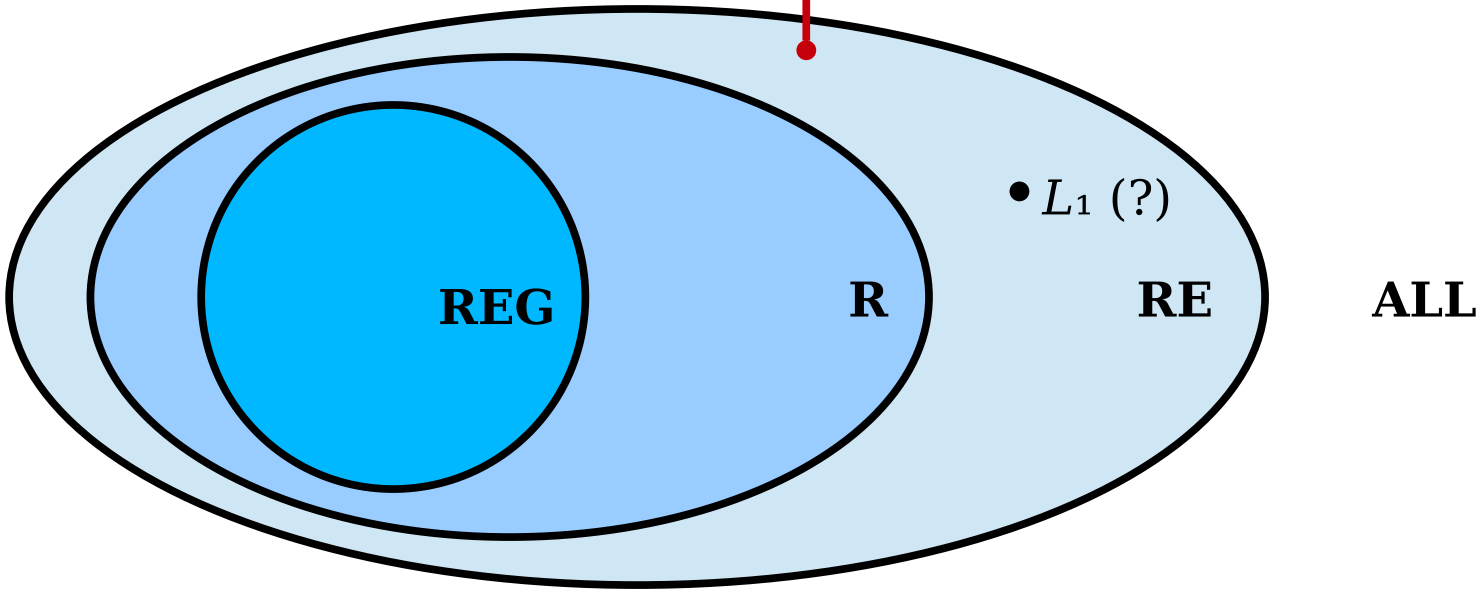

The next step is to determine whether it's also in class **R**.

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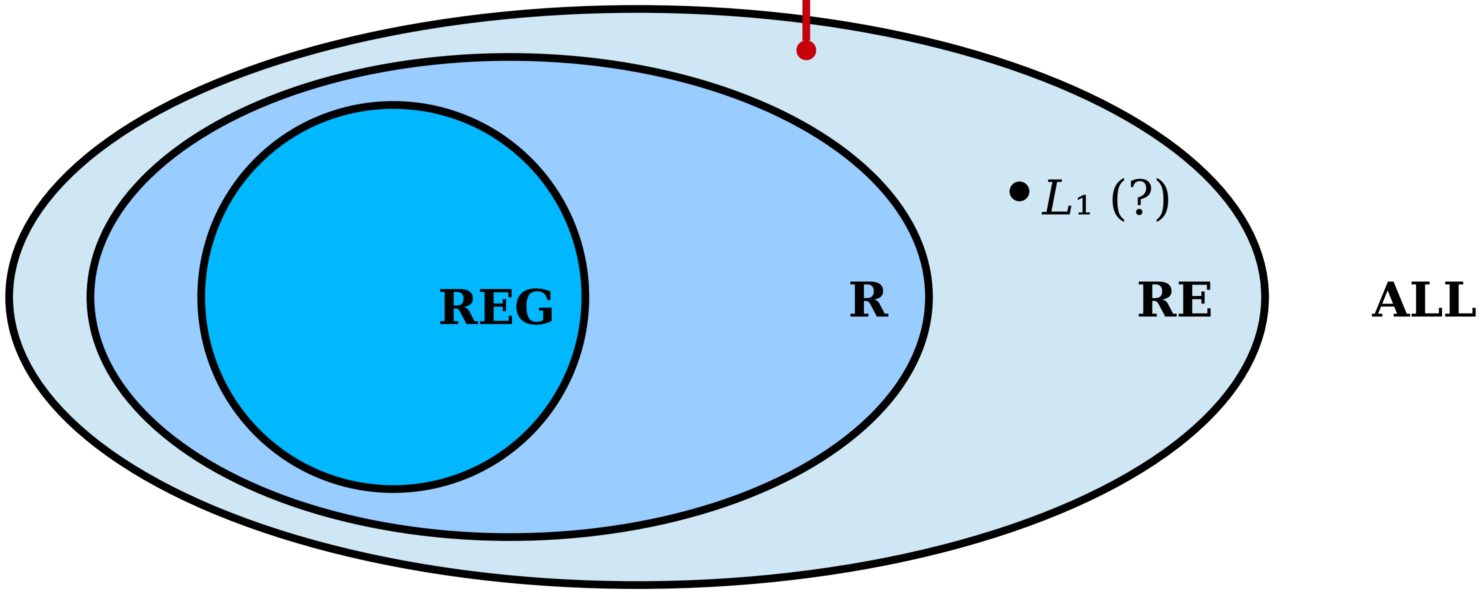

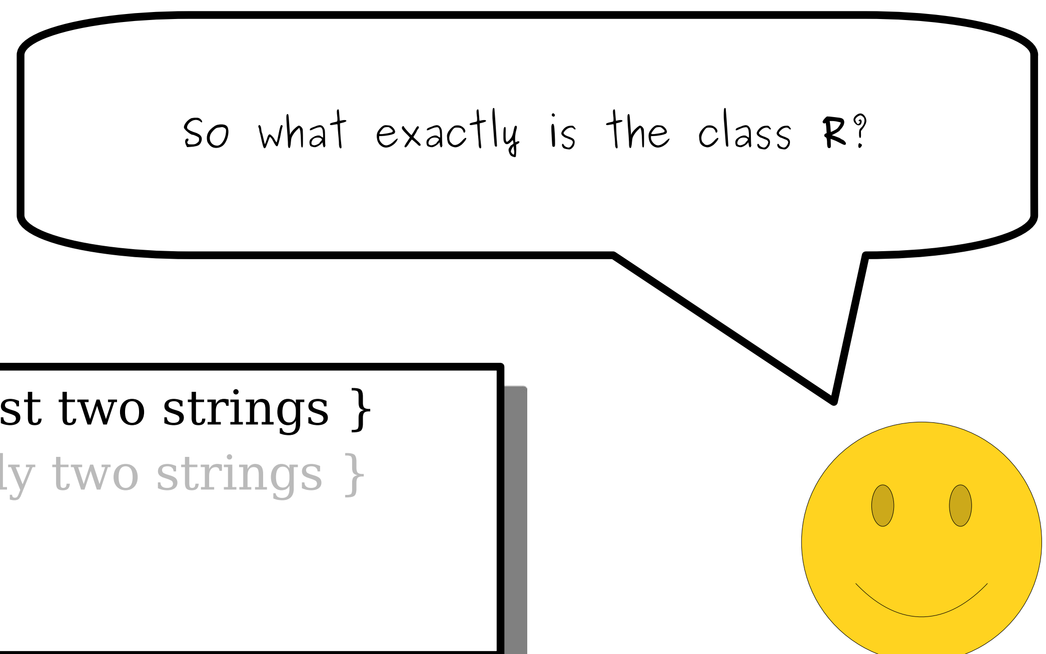

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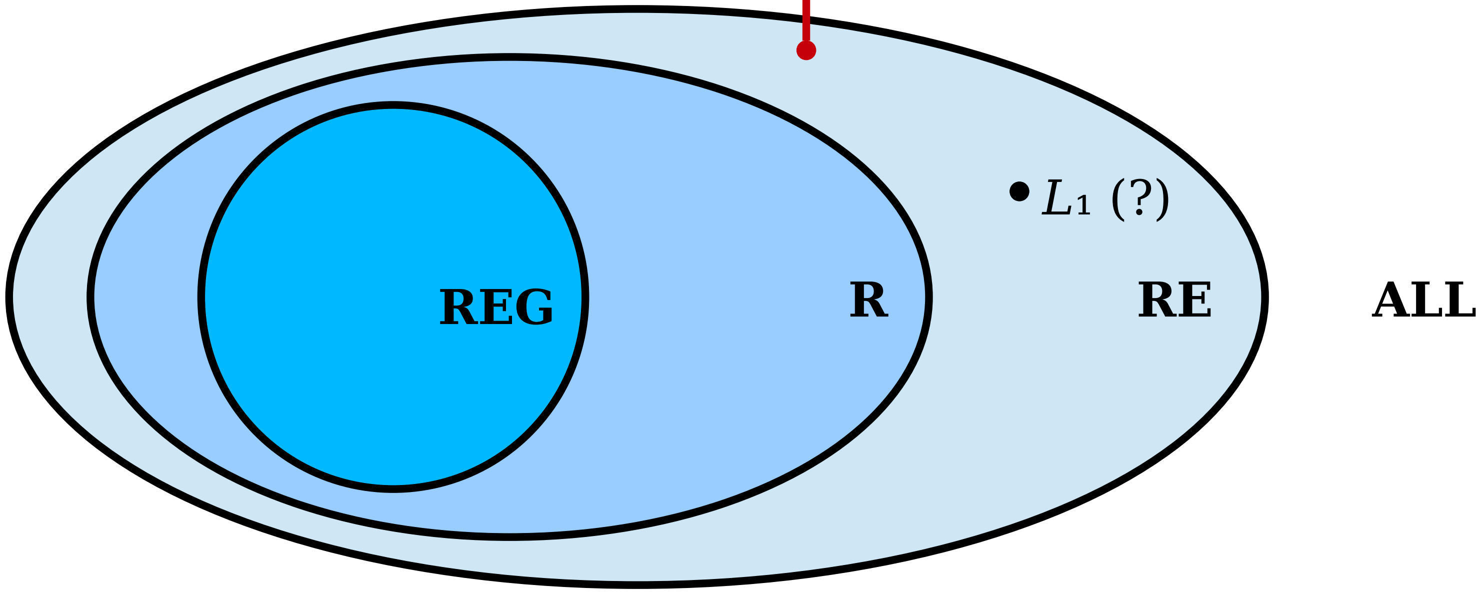

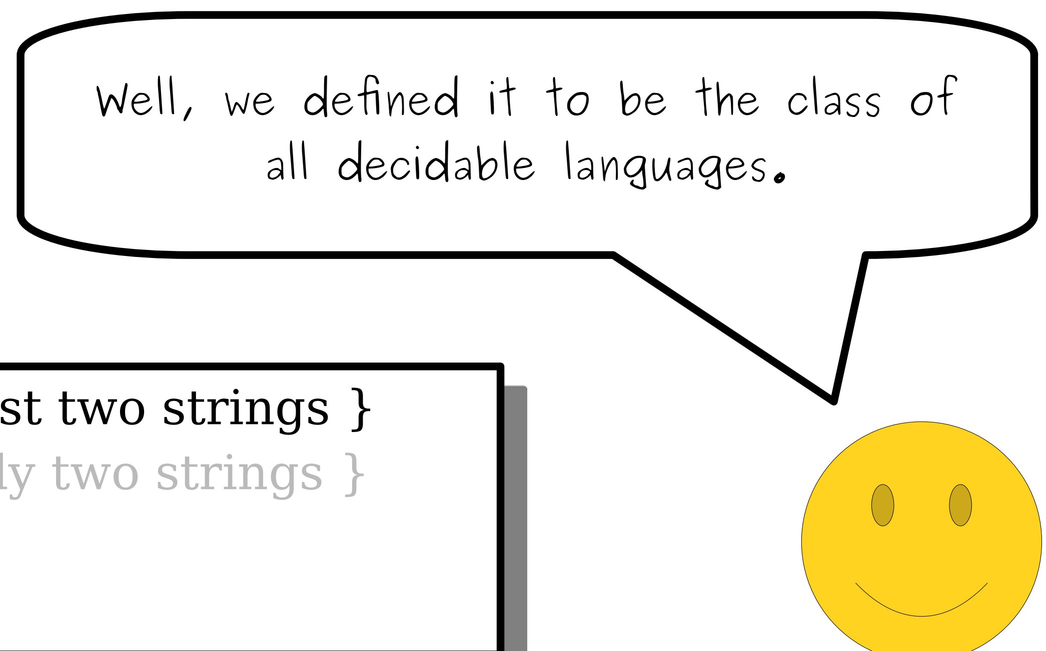

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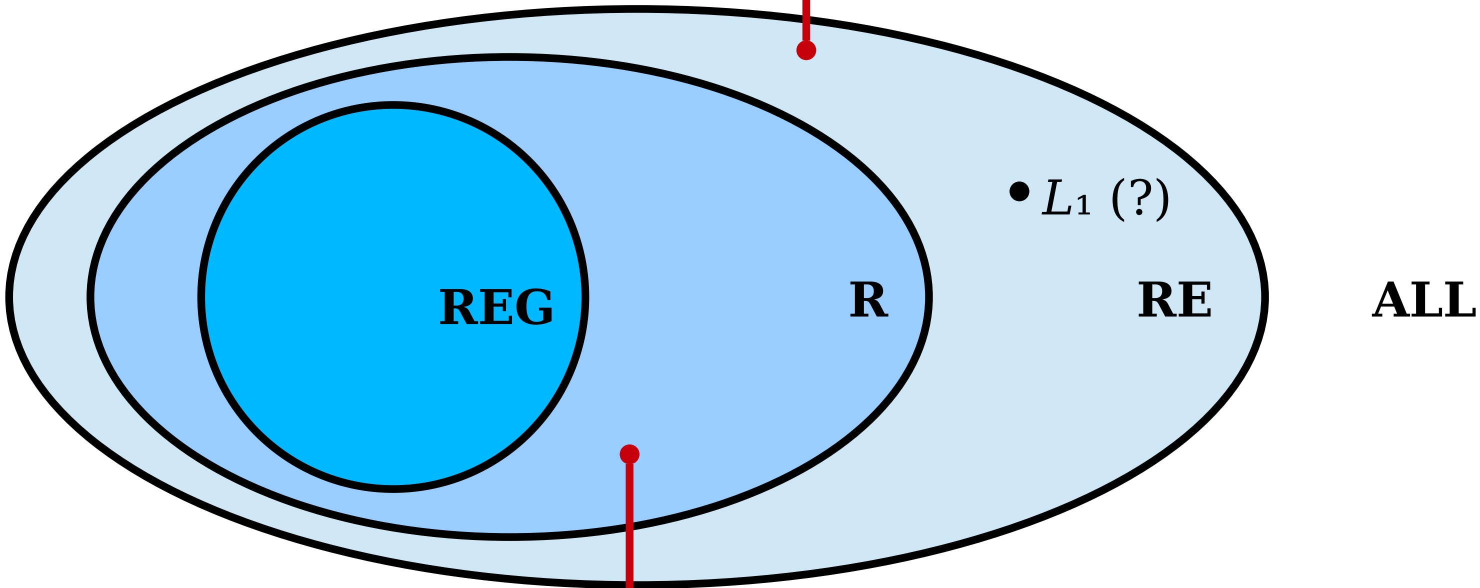

R: Languages with Deciders

That means that it's the class of all languages that have deciders.

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RE: Languages with Verifiers

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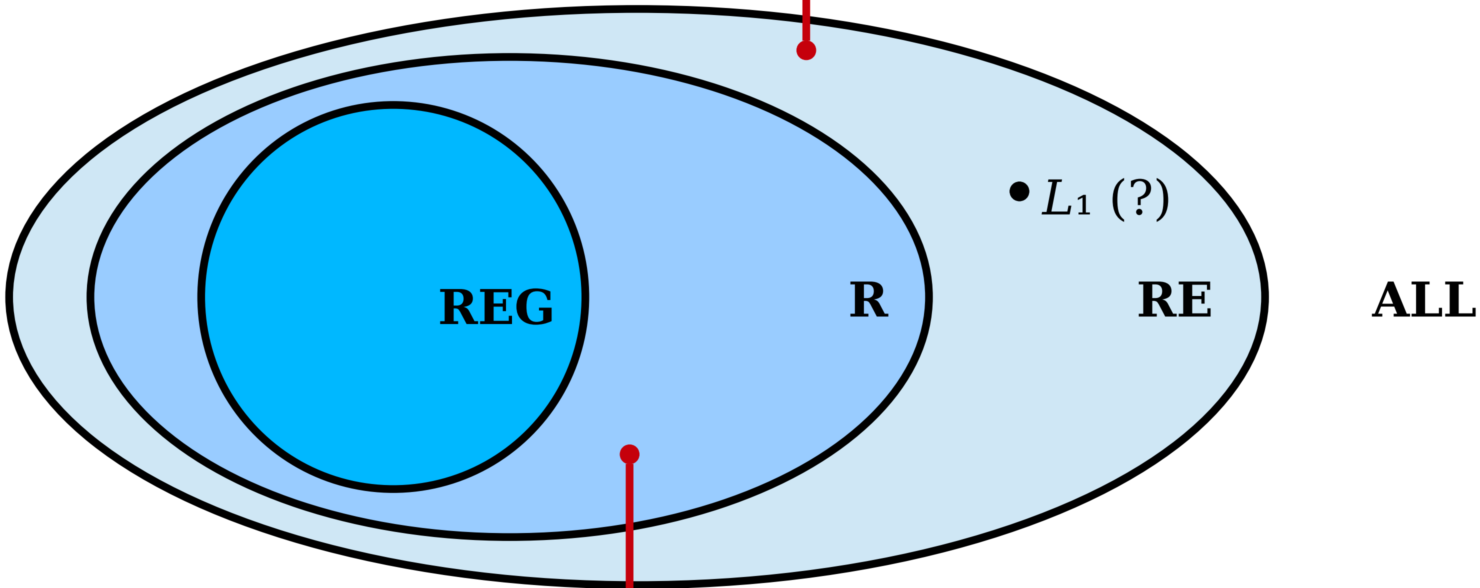
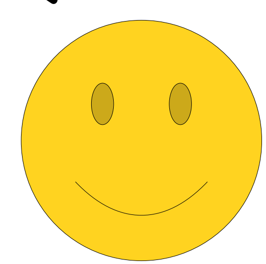


R: Languages with Deciders

You can reason about whether a language belongs to class R by thinking about whether you could build a decider for it.



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RE: Languages with Verifiers

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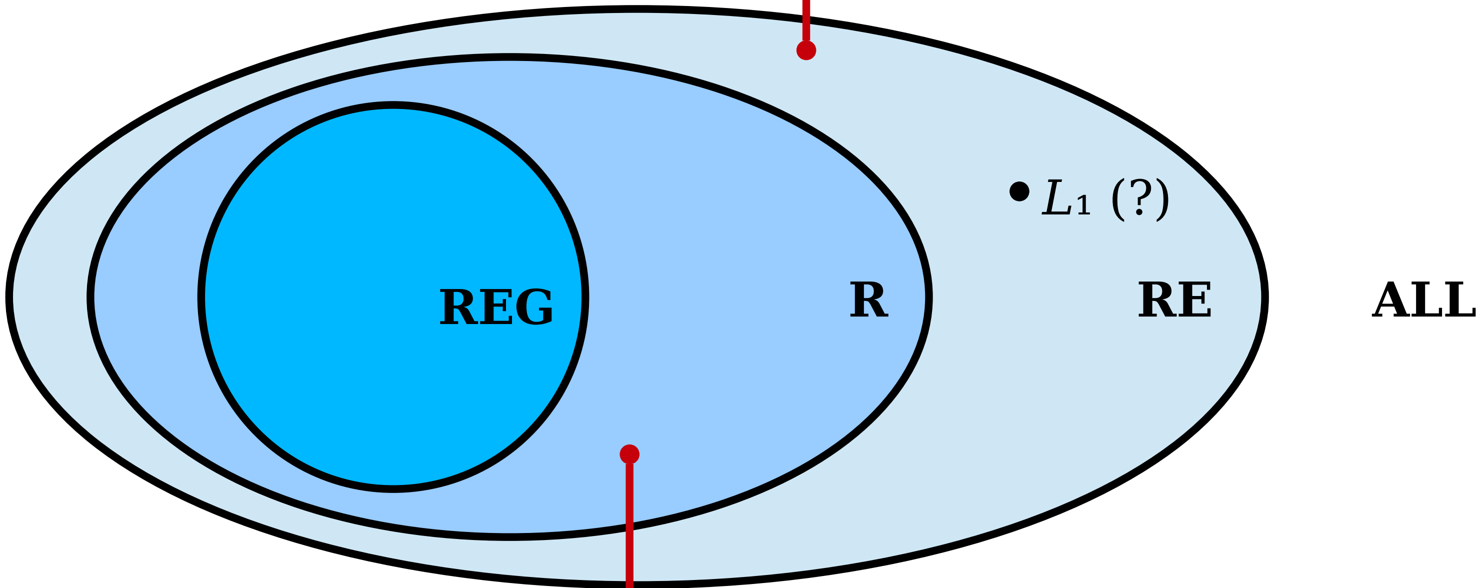
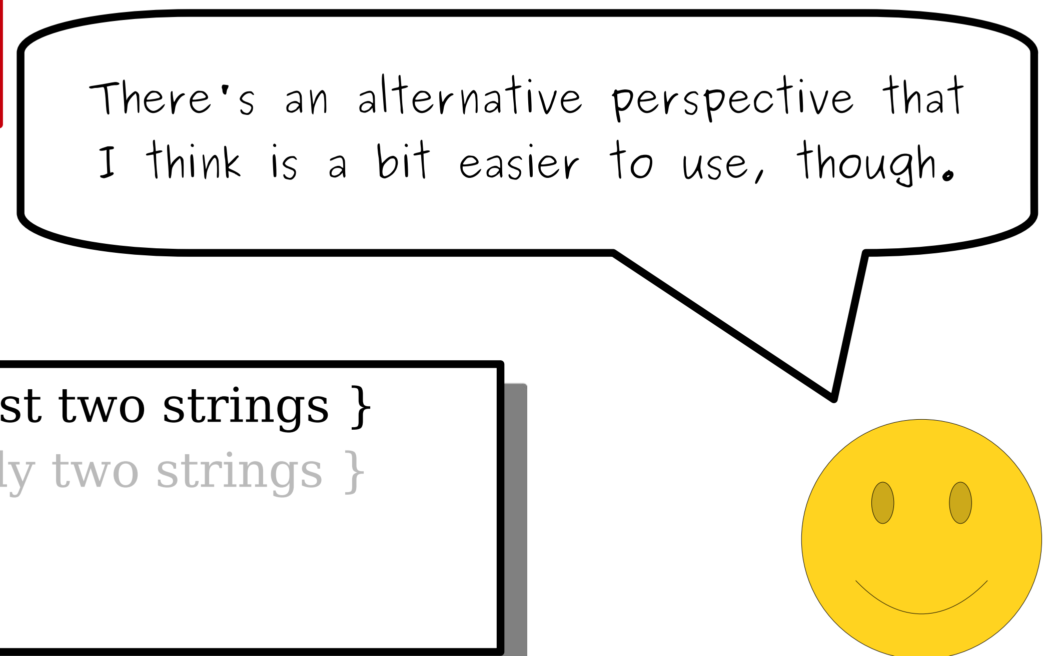


R: Languages with Deciders



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RE: Languages with Verifiers

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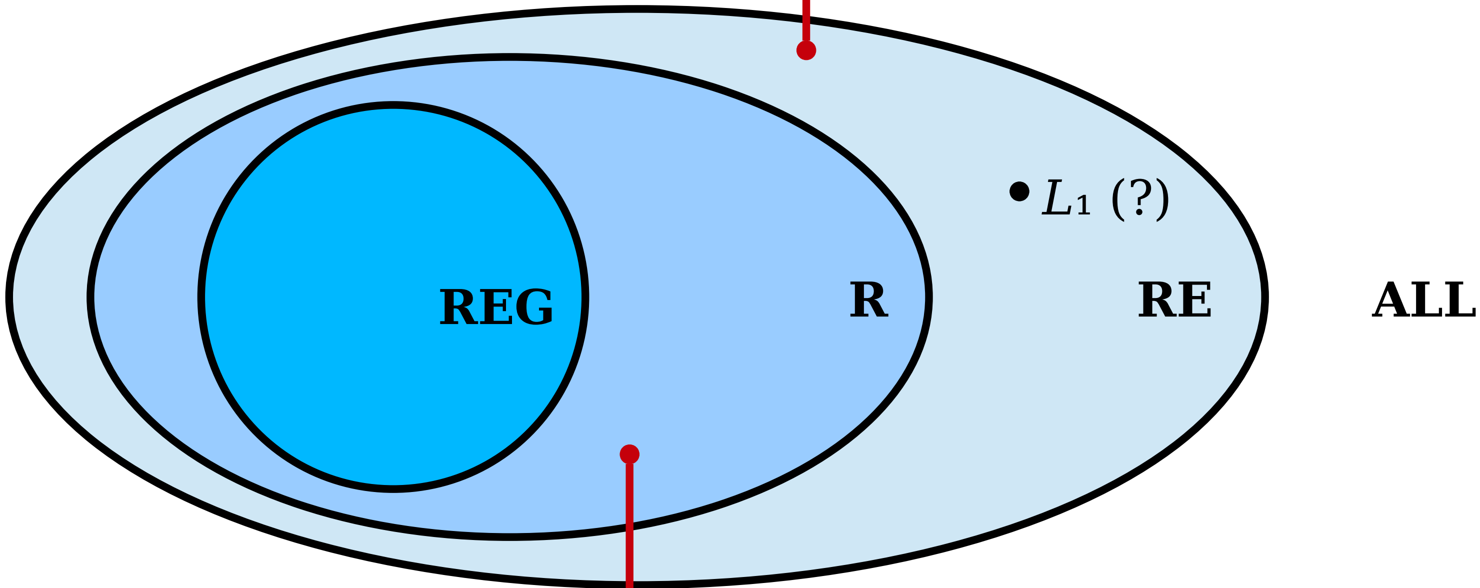

R: Languages with Deciders

On Problem Set Nine, there's a problem entitled "Double Verification."

L_1 = { ⟨M⟩ | M is a TM that accepts at least two strings }

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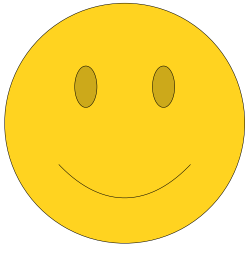

RE: Languages with Verifiers

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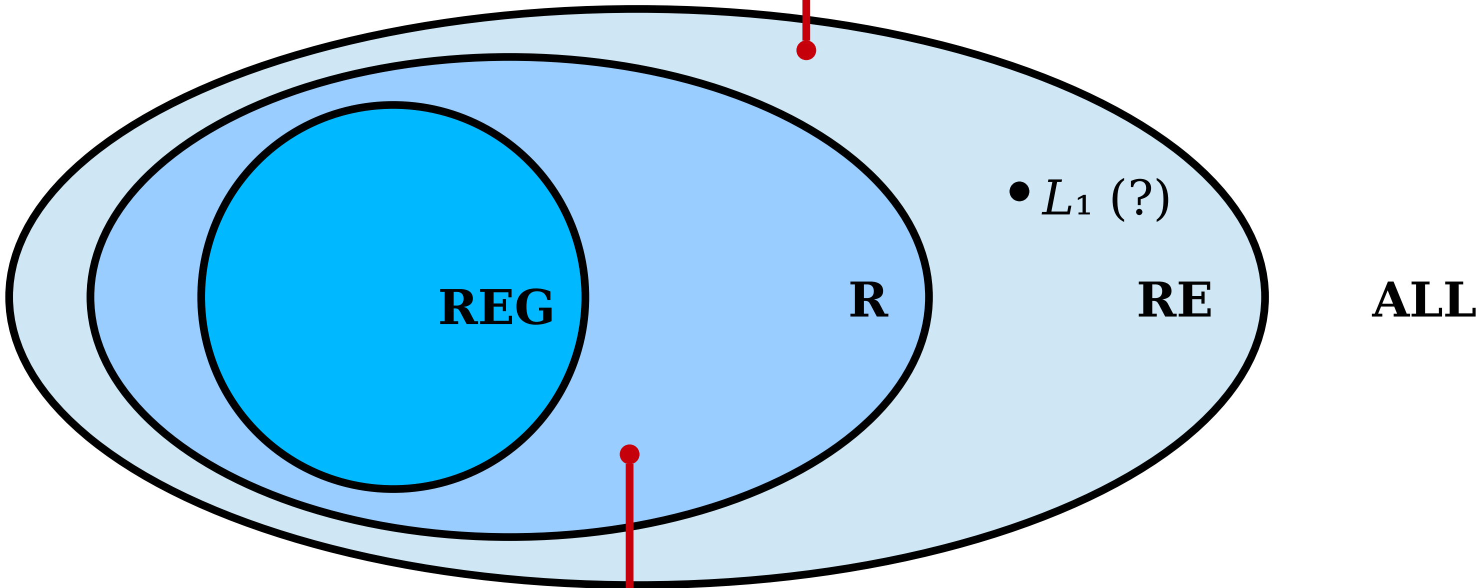

R: Languages with Deciders

We asked you to prove this statement:

If $L \in$ **RE** and $\overline{L} \in$ **RE**, then $L \in$ **R**.

What exactly does that mean?

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RE: Languages with Verifiers

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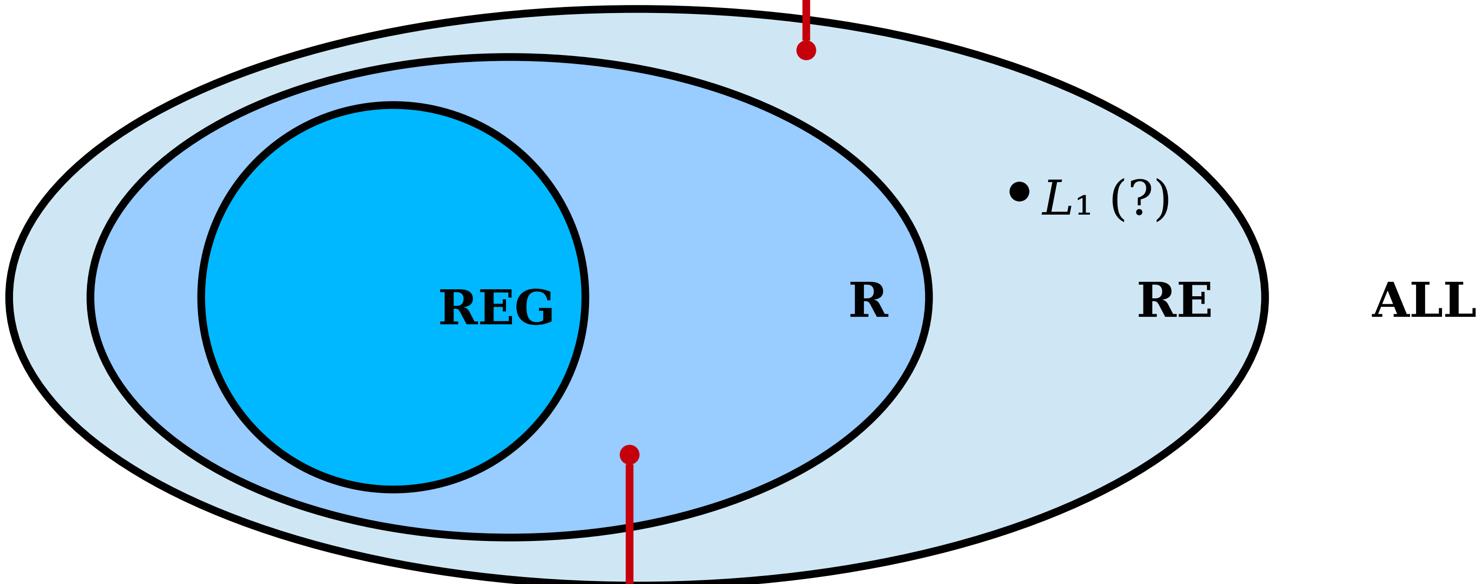

R: Languages with Deciders

If $L \in$ **RE** and $\overline{L} \in$ **RE**, then $L \in$ **R**.

From what we've talked about so far, you probably have a slightly better feel for what it means for L to be in **RE**.

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RE: Languages with Verifiers

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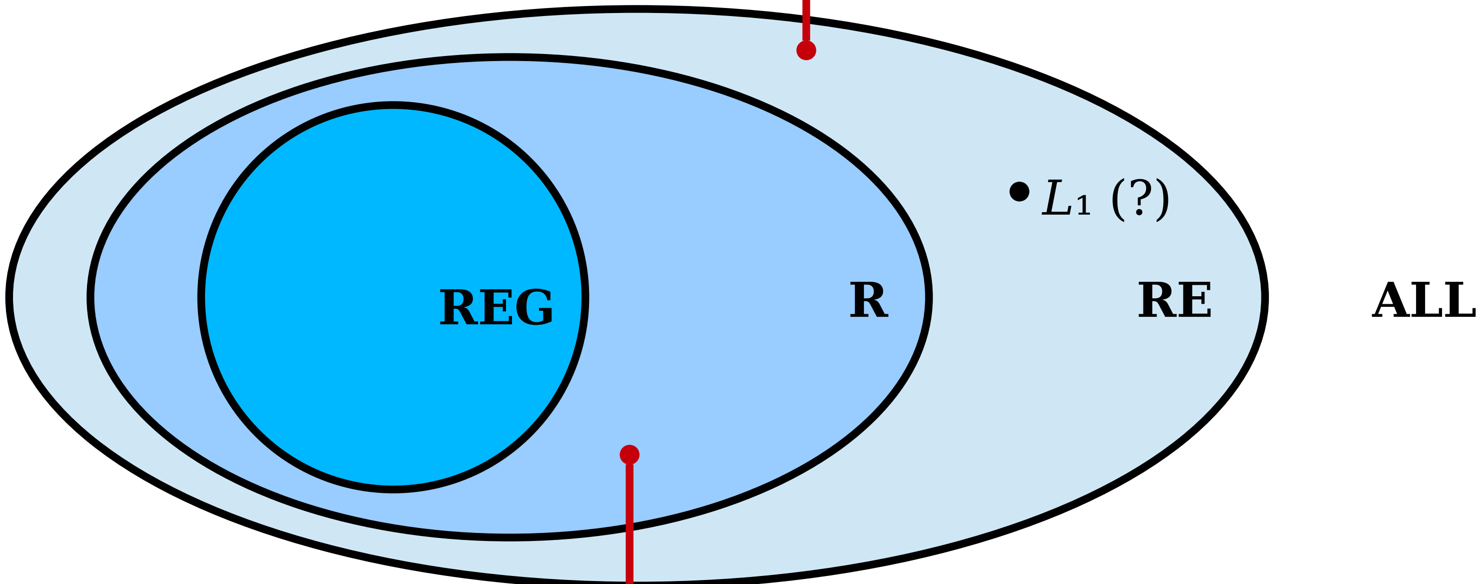

R: Languages with Deciders

If $L \in$ **RE** and $\overline{L} \in$ **RE**, then $L \in$ **R**.

But what exactly does it mean for the <u>complement</u> of L to be in **RE**?

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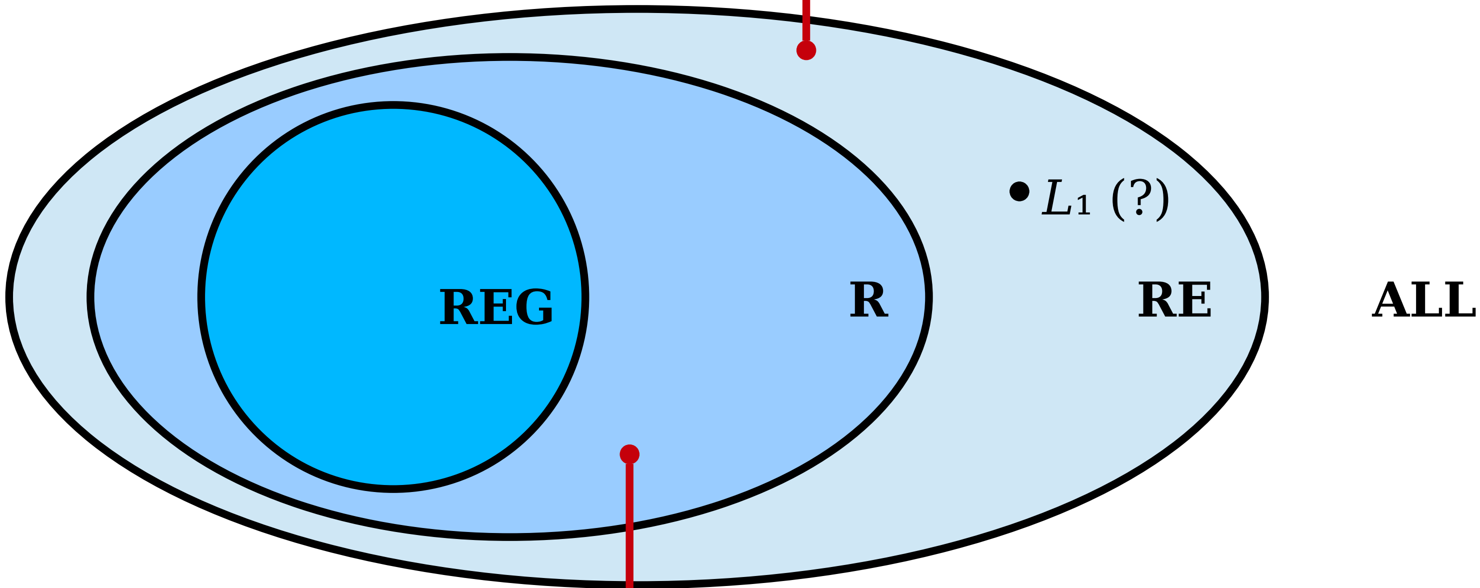
RE: Languages with Verifiers

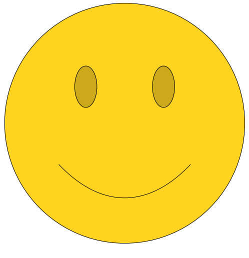
Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

If $L \in$ **RE** and $\overline{L} \in$ **RE**, then $L \in$ **R**.

Going off of our proof-based intuition, if the complement of L is in **RE**, it means that given any string w that is <u>not</u> in L, there's a way to prove it's not in L.



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RE: Languages with Verifiers

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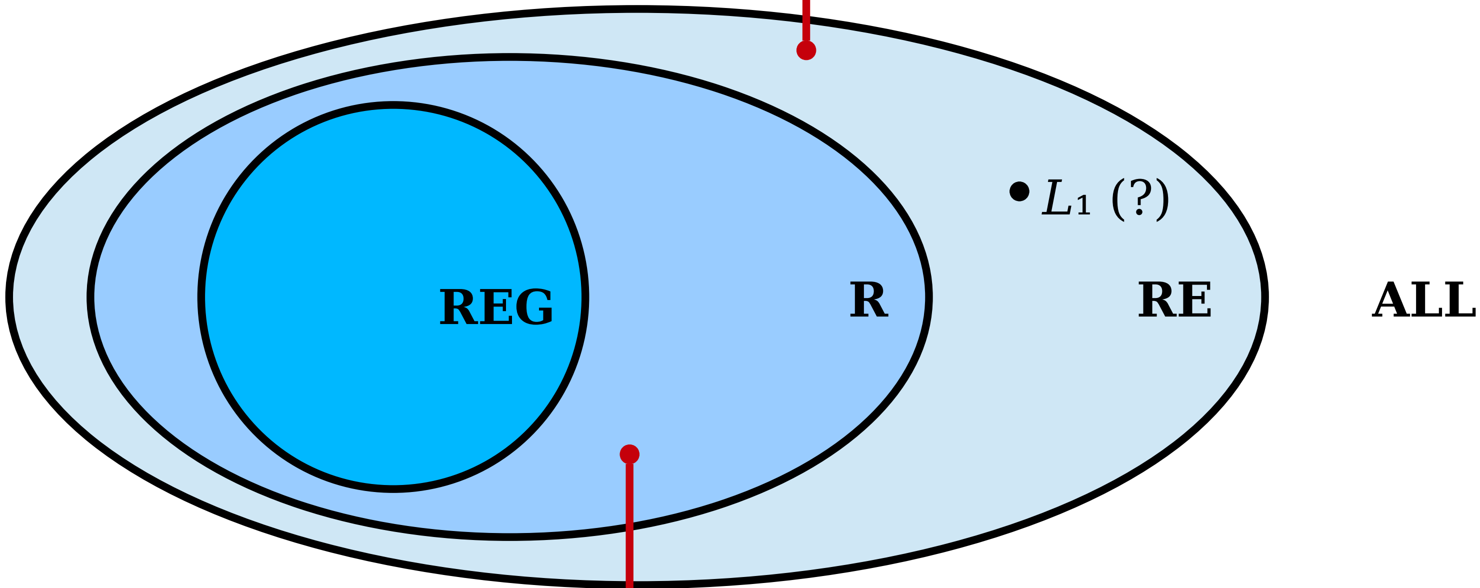

R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

This turns out to be a great way of intuiting the class **R**. A language belongs to **R** if it's in **RE**, and for any string that isn't in the language, there's a way to prove it's not in the language.

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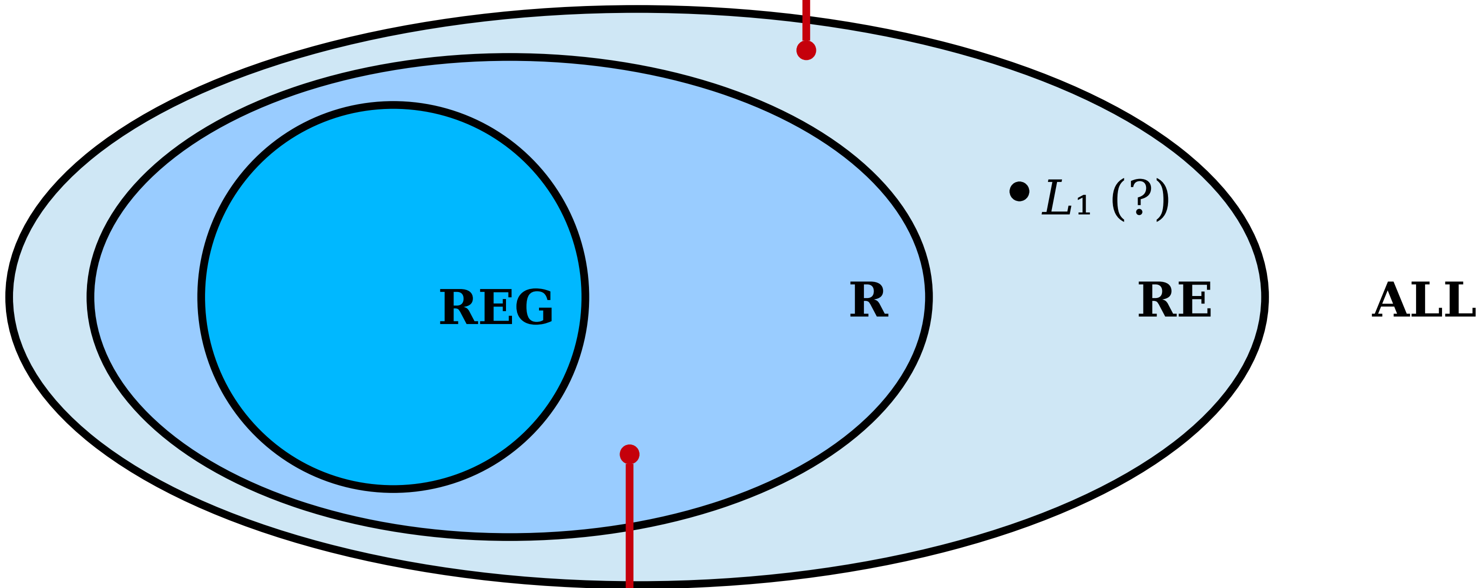
RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

(Although we only had you prove the forward direction of the implication in the Double Verification problem, turns out the reverse direction holds as well. This gives an exact characterization of **R**!)

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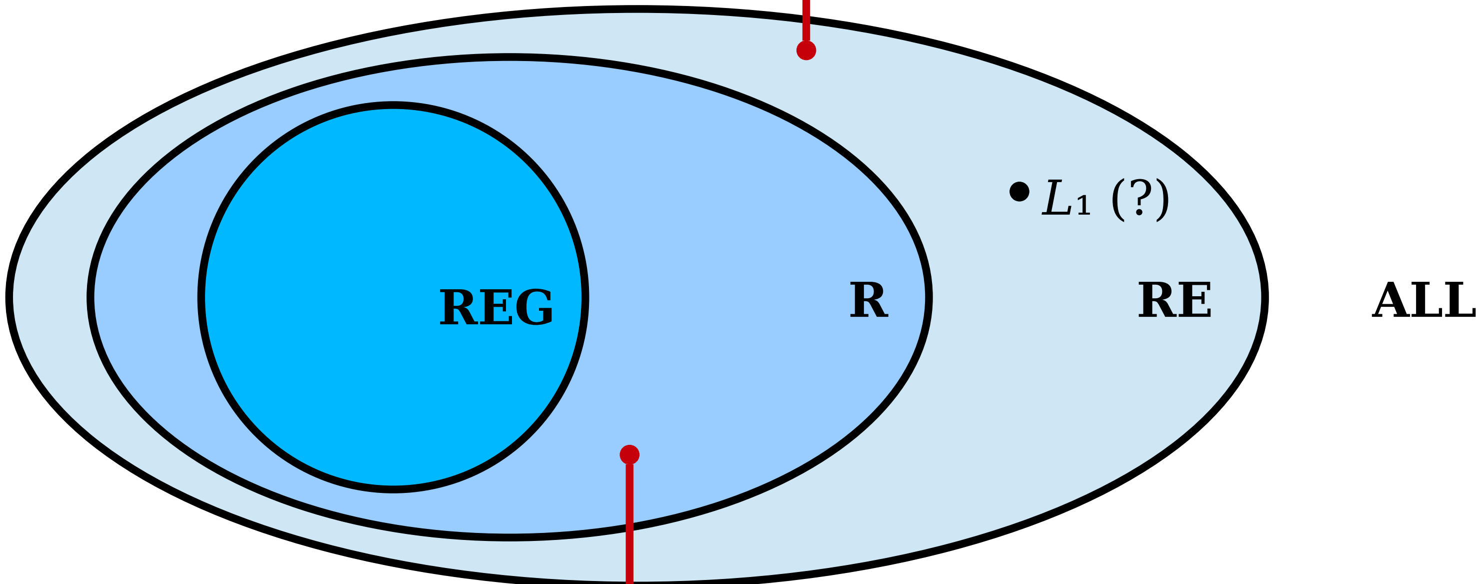
RE: Languages with Verifiers

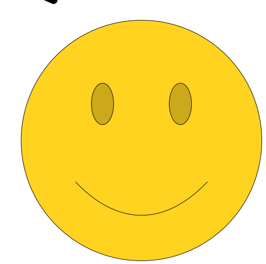
Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

Now, let's jump back to our particular language L_1 here and use this intuition to think about whether or not it belongs to class **R**.



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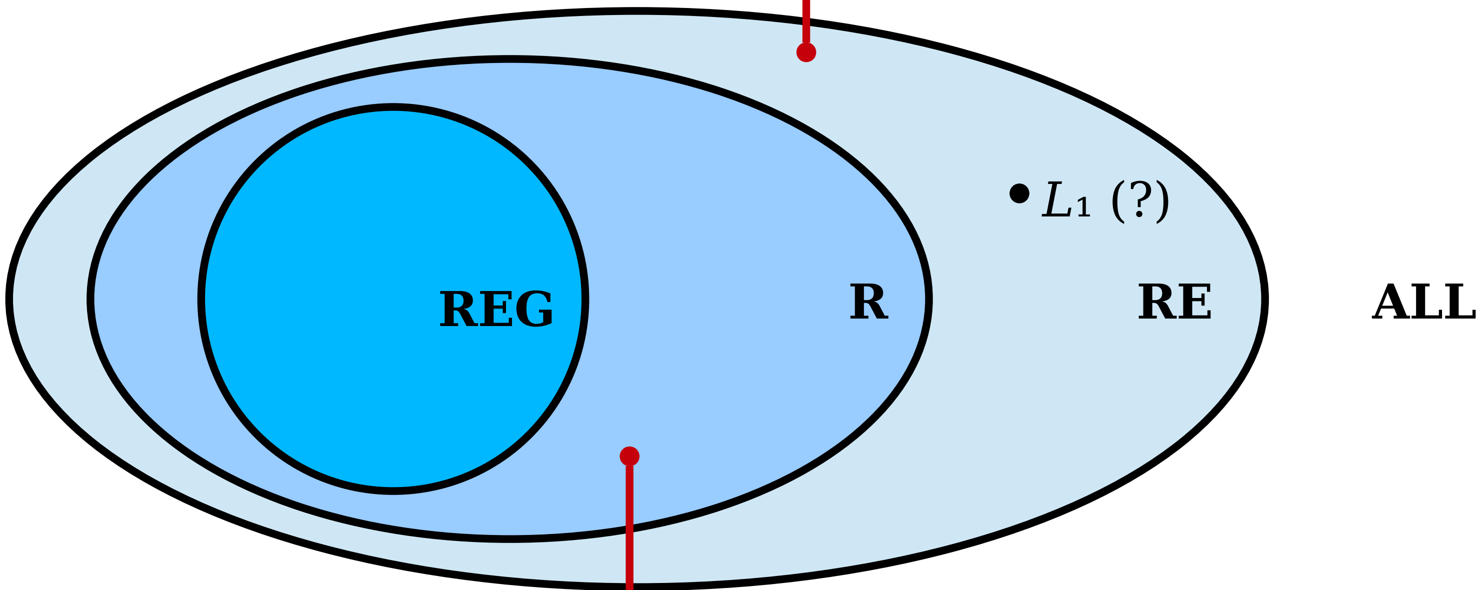
RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

So imagine that you have some string that isn't in L_1.

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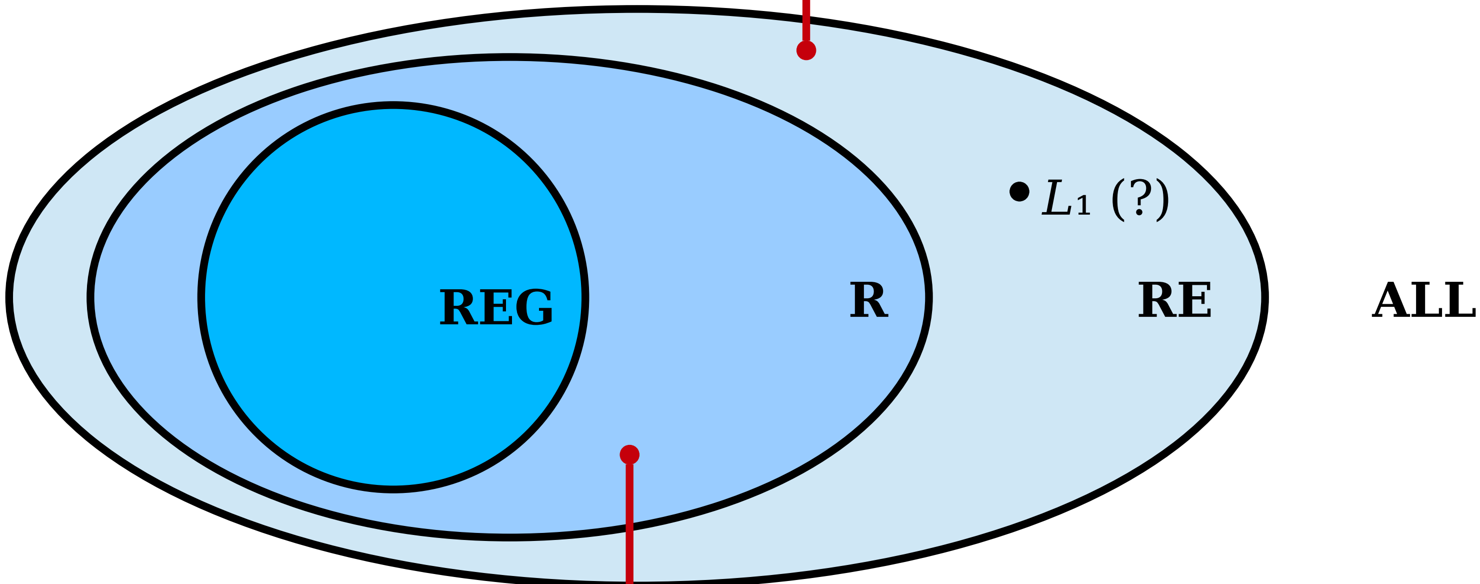

R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

In other words, imagine you have TM M where it's not the case that M accepts at least two strings.

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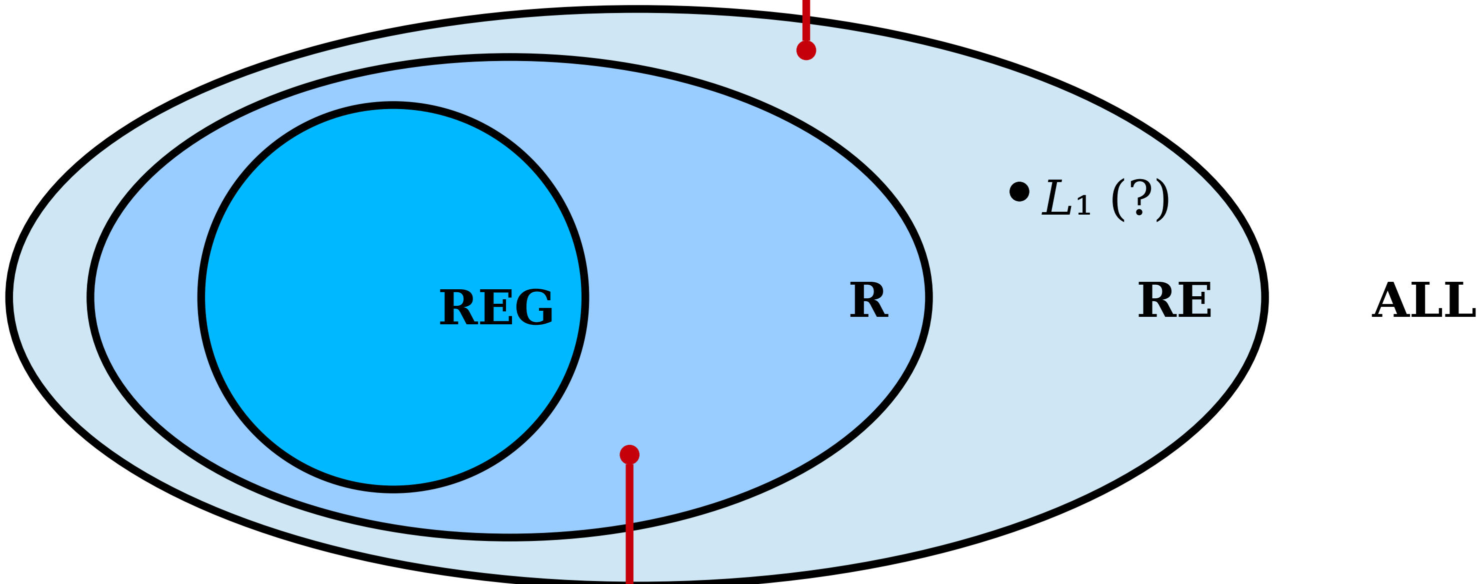
RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

That means that M must accept either no strings at all or just one string.

(Do you see why?)

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RE: Languages with Verifiers

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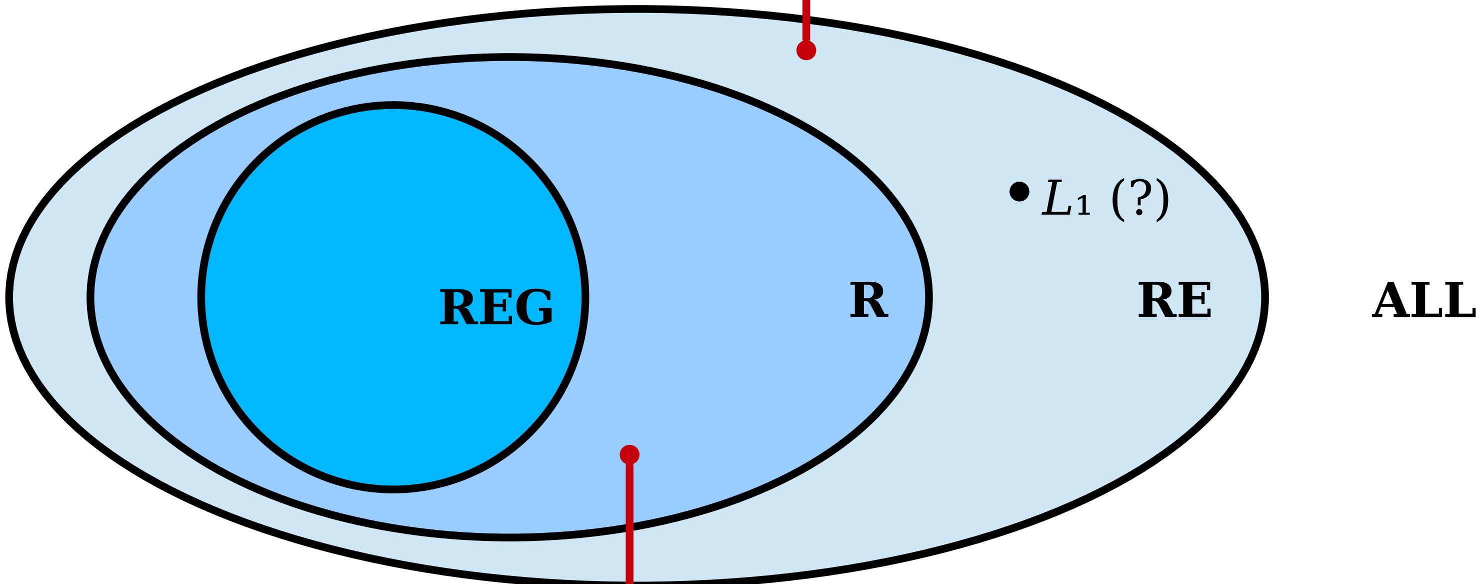

R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

So now the question is the following: if you have a TM that accepts either no strings or just one string, could you prove it to someone who was skeptical but honest?

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RE: Languages with Verifiers

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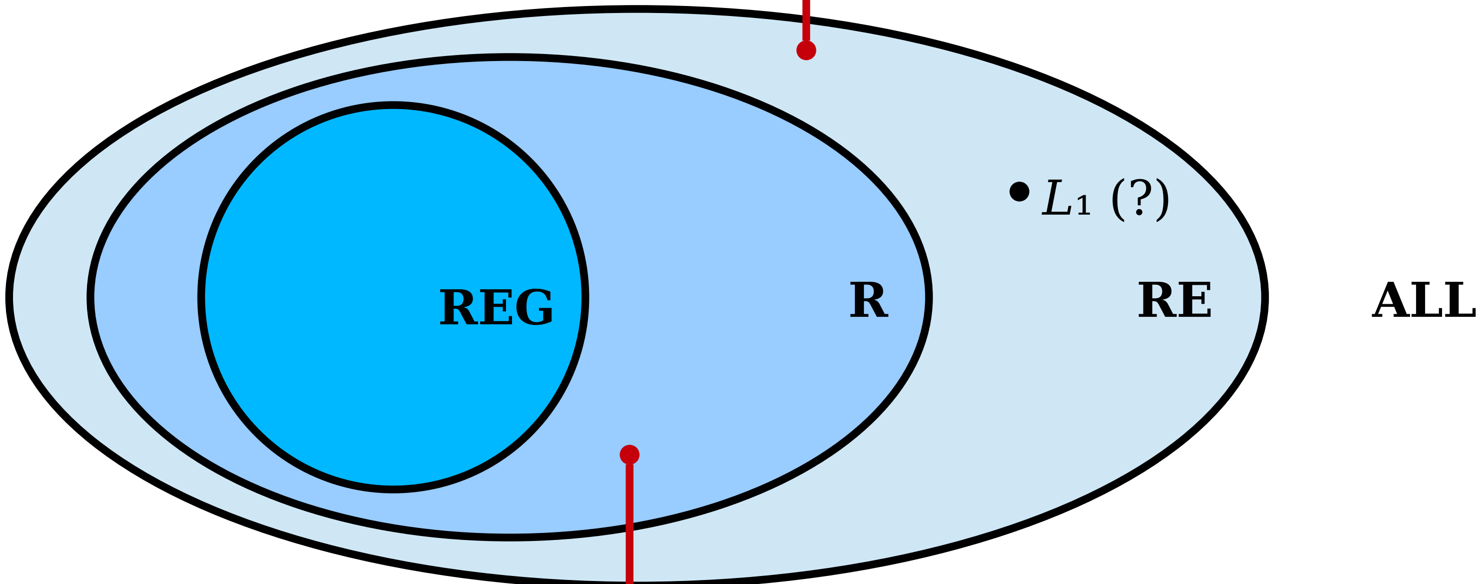

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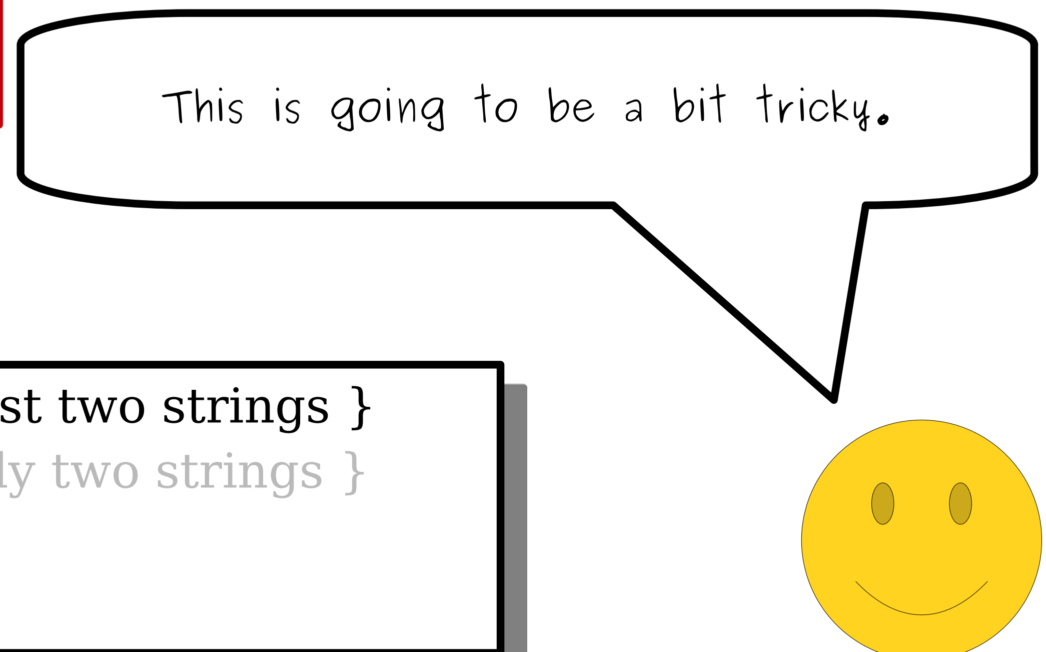

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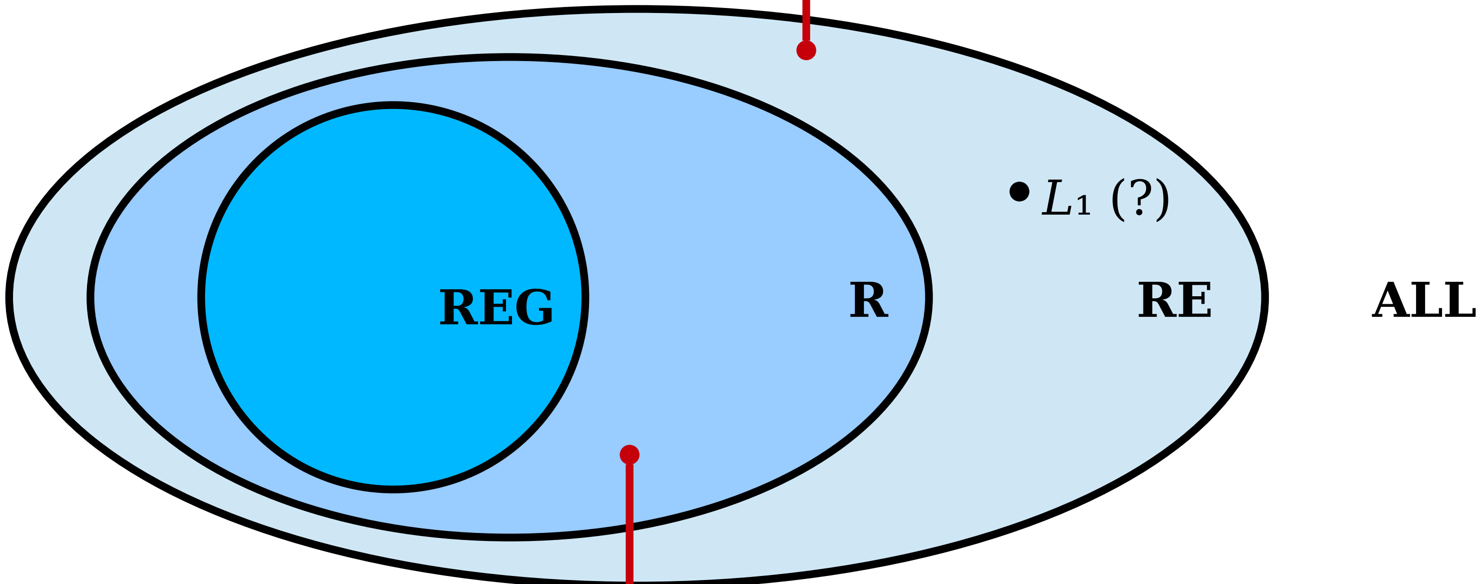

R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

If you want to convince someone that M only accepts at most one string, you need to convince them that out of the infinitely many strings that are out there, the TM accepts at most one.

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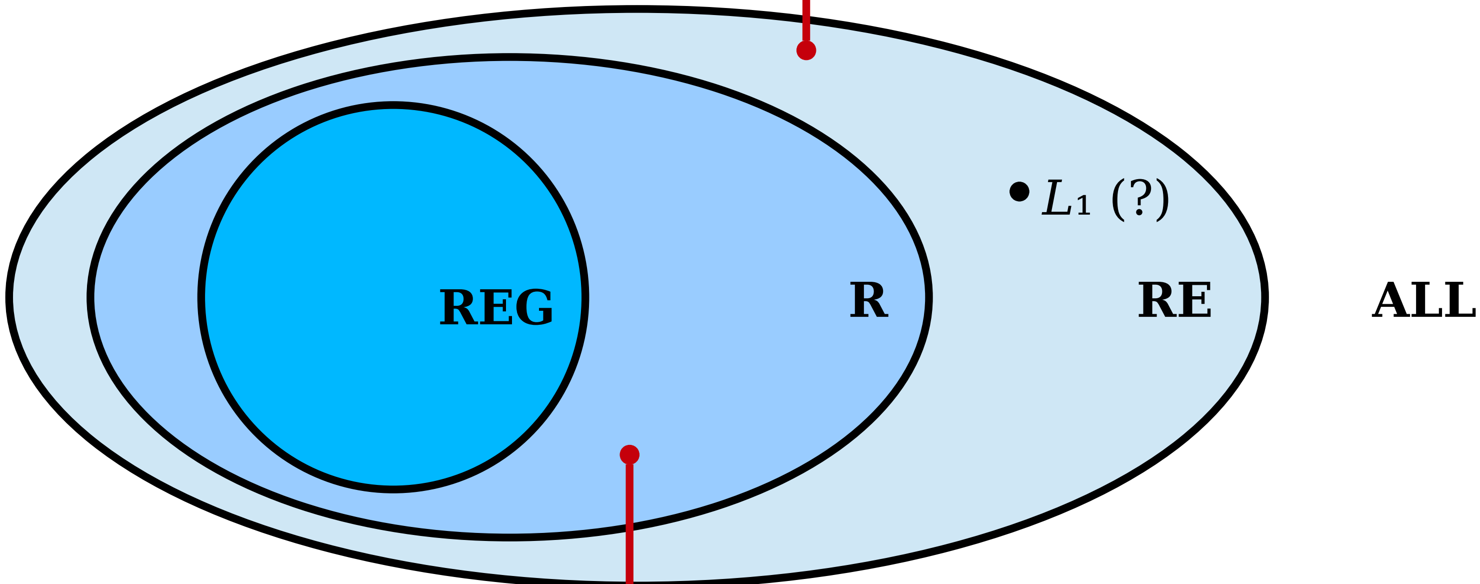

R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

As we've seen before, though, we know that the only general way to find out what a TM will do on a string is to run the TM on that string and see what happens.

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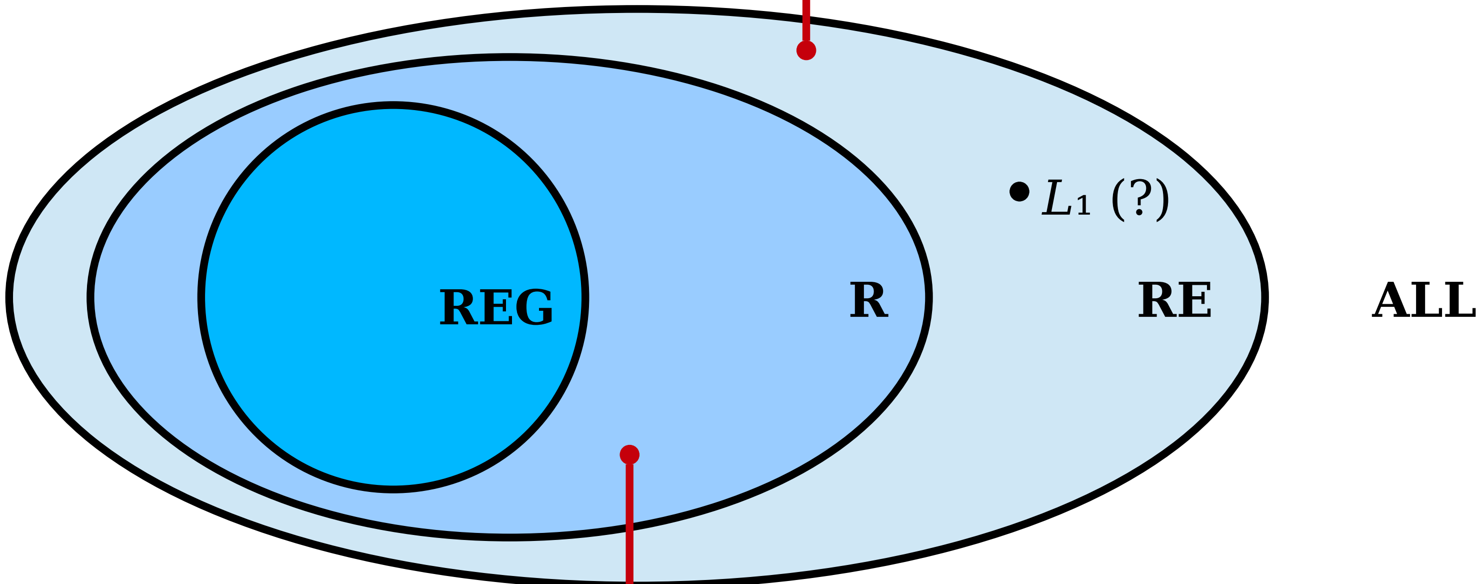

R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

So if we want to convince someone that a TM doesn't accept infinitely many different strings, we're out of luck! In the general case, we'd have to run the TM on all those strings…

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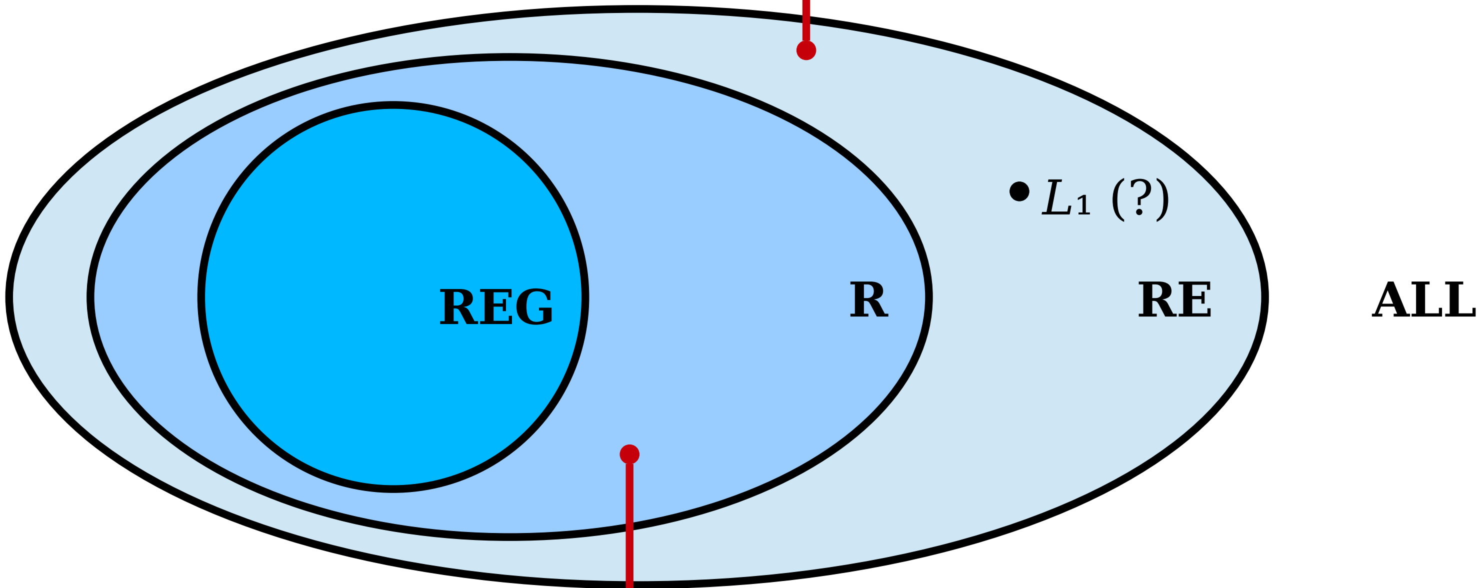

R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

… and given that there are infinitely many of them, we'll never finish checking them all.

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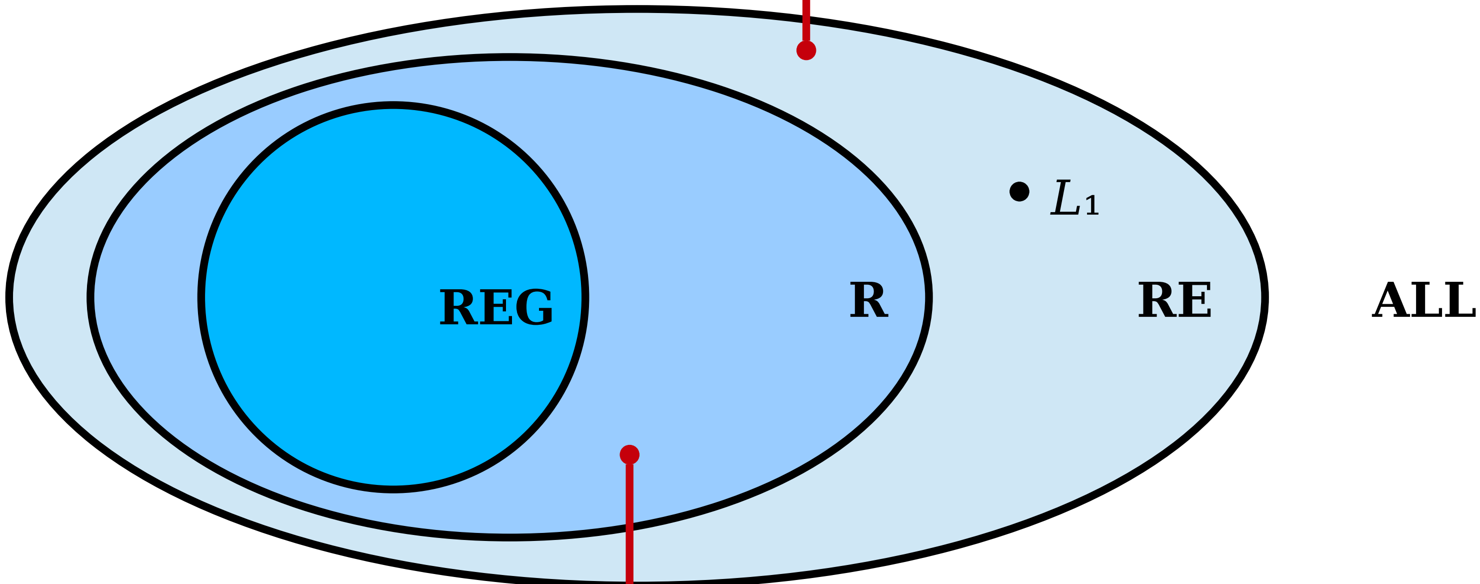
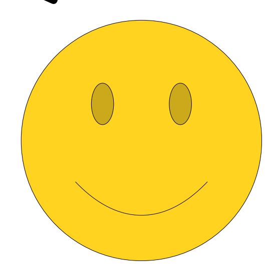


R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

So, based on the intuition that a language is in R if we can always prove it when strings aren't in the language, we'd suspect that this language is not in R.



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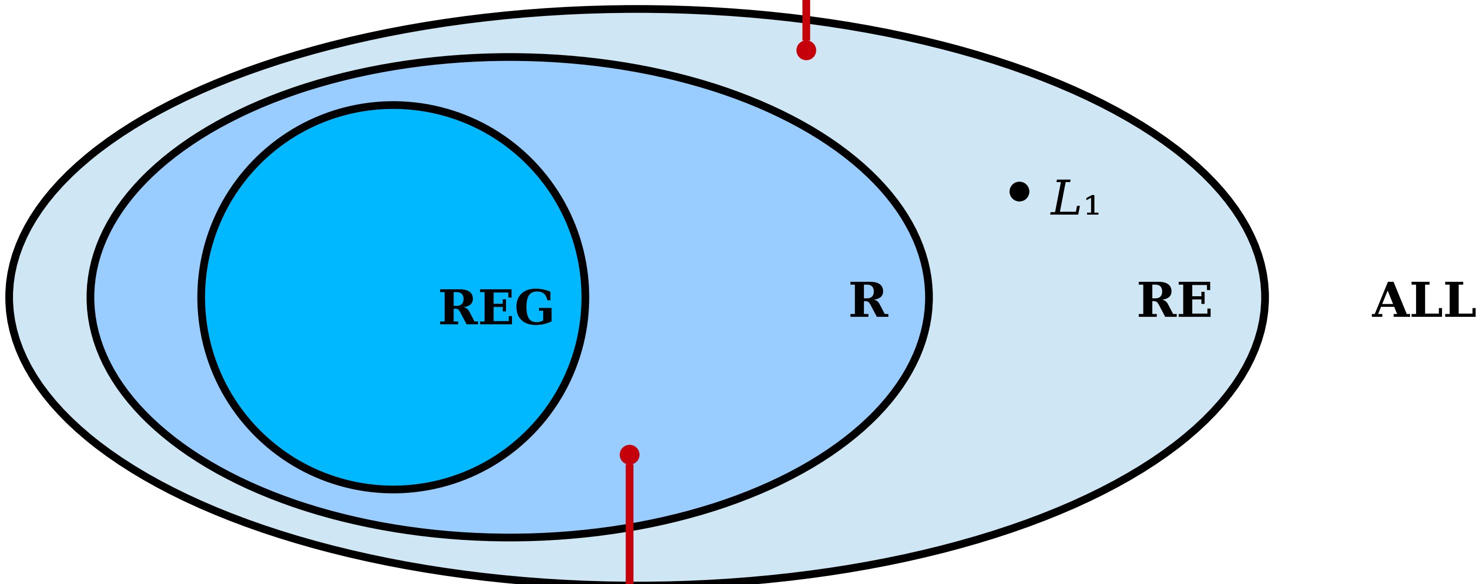
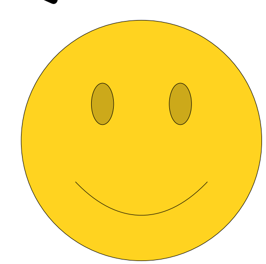


R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

To actually go and prove this, we could use some kind of self-reference trick and build a machine that asks whether it's going to accept at least two strings, then does the opposite.



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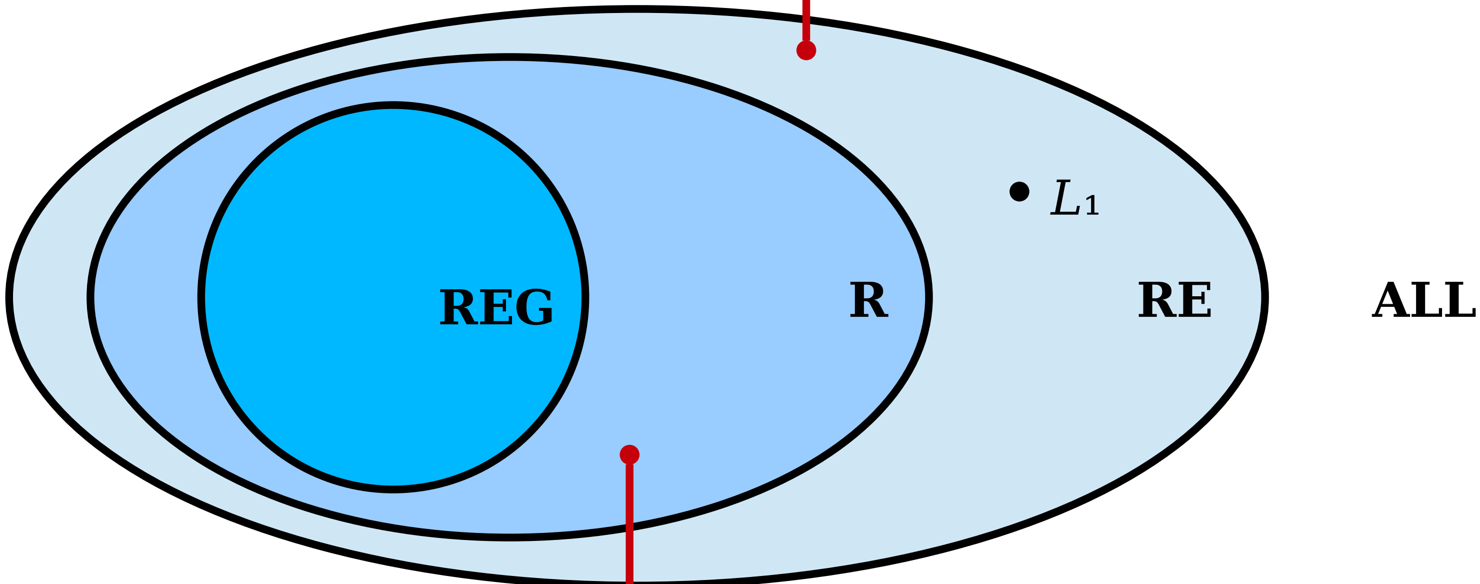
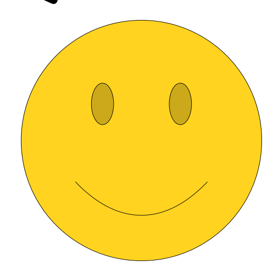


R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

In fact, that's such a good exercise that you should stop reading this and go do it right now. The Guide to Self-Reference might help you there.



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RE: Languages with Verifiers

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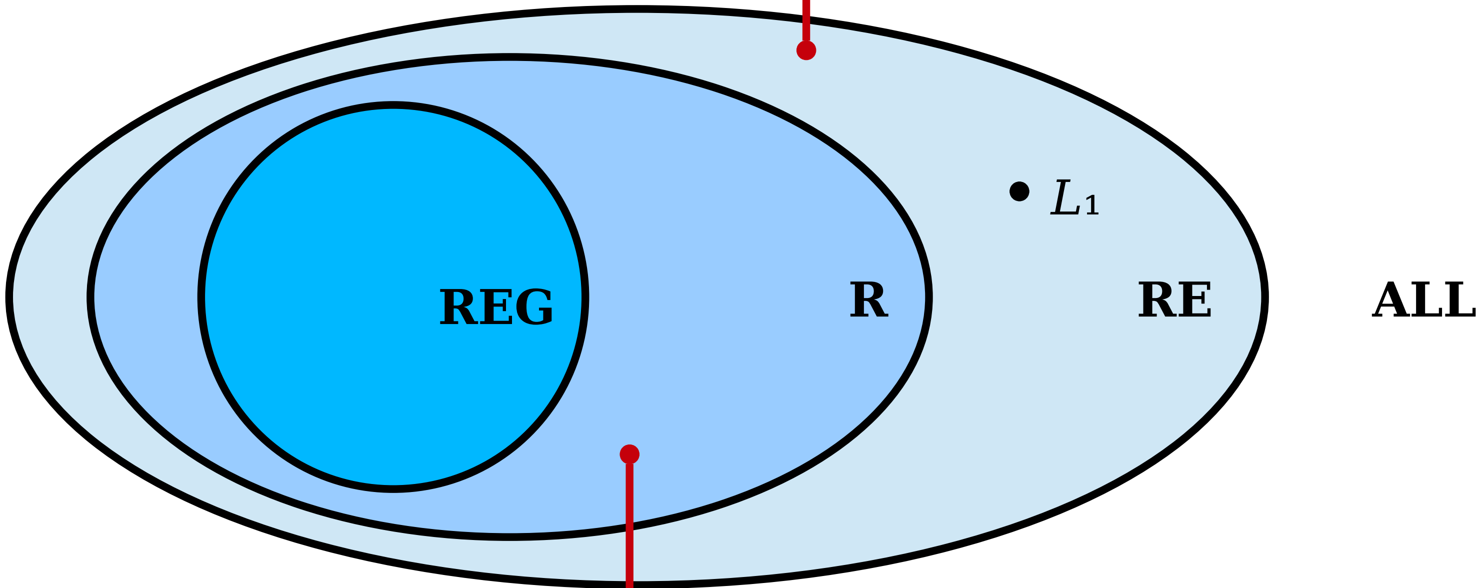

R: Languages with Deciders

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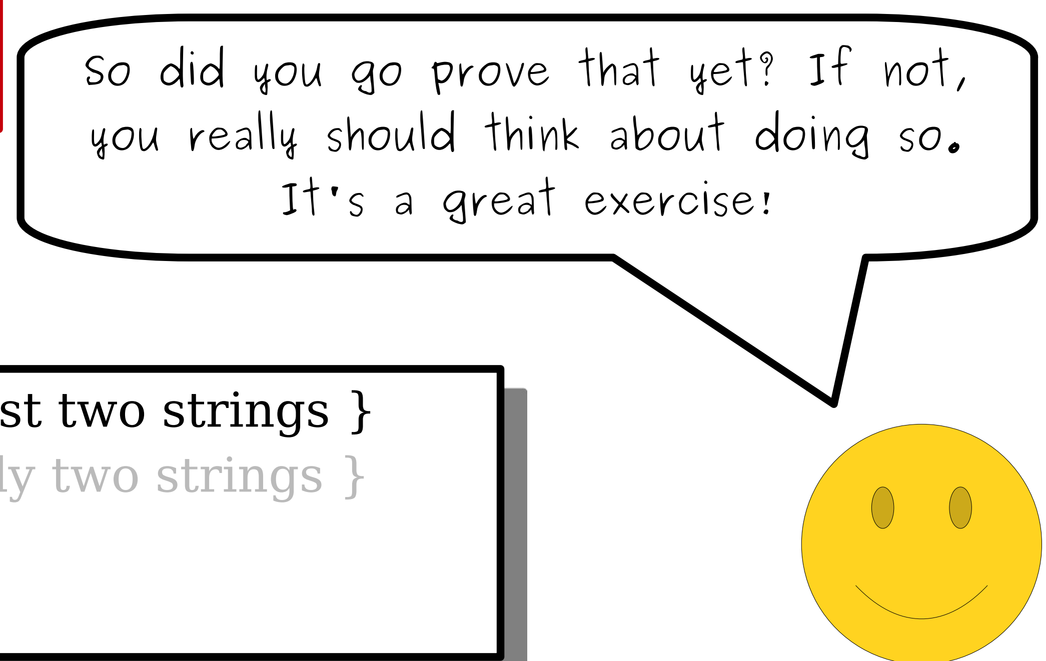

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RE: Languages with Verifiers

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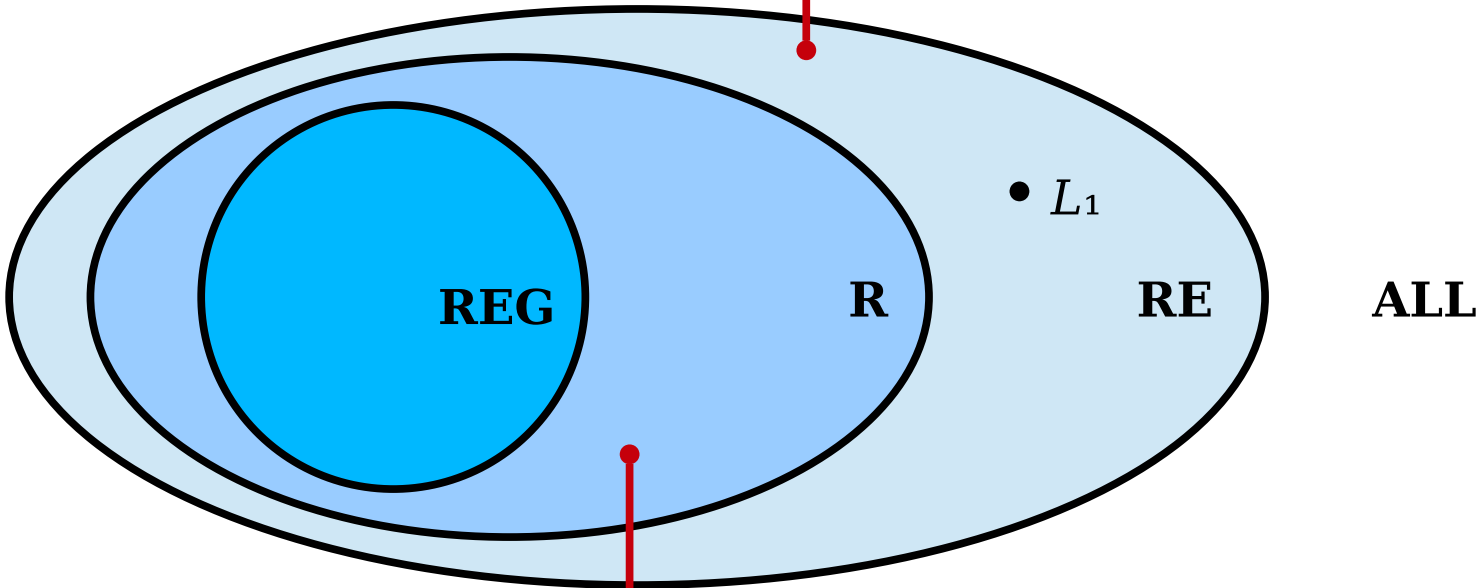

R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

So at this point we've got this language settled in the right place. It's in **RE**, but it's not in **R**.

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L_2 = { ⟨M⟩ | M is a TM that accepts exactly two strings }

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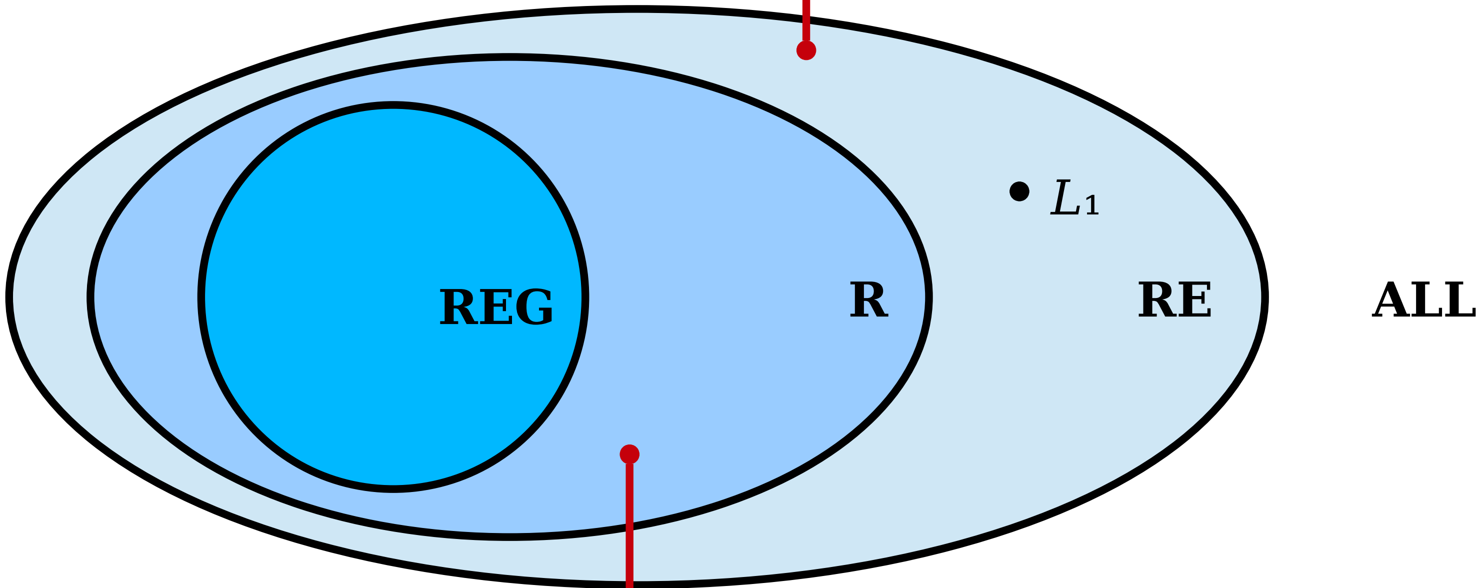
L_4 = { $\mathtt{a}^n\mathtt{b}^n$ | $n \in \mathbb{N}$ and $n \leq 1000$ }

RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

Before we move on to the next language, I wanted to take a minute to address a common question we get on problems like these.

$L_1 = \{ \langle M \rangle \mid M$ is a TM that accepts at least two strings $\}$

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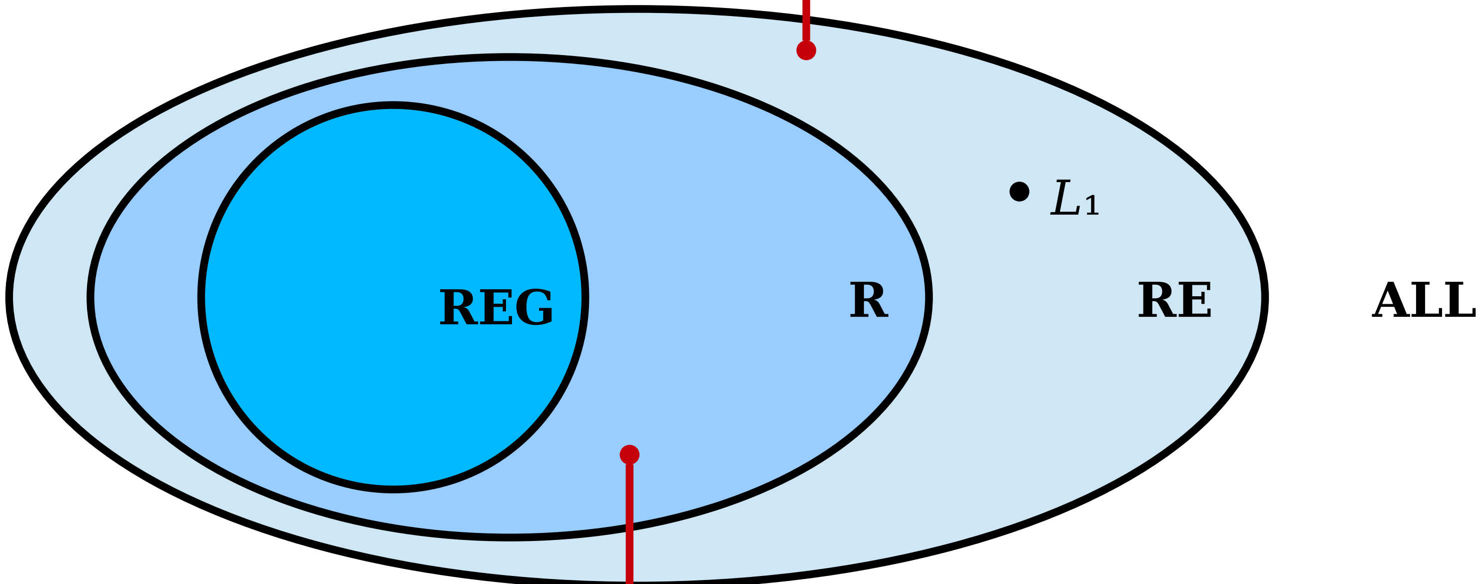
$L_3 = \{ \mathtt{a}^n\mathtt{b}^n \mid n \in \mathbb{N}$ and $n > 1000 \}$

$L_4 = \{ \mathtt{a}^n\mathtt{b}^n \mid n \in \mathbb{N}$ and $n \leq 1000 \}$

RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

If you look at the description of the language, you can see that it says something about TMs that accept at least two strings.

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$L_3 = \{ \mathbf{a}^n\mathbf{b}^n \mid n \in \mathbb{N}$ and $n > 1000 \}$

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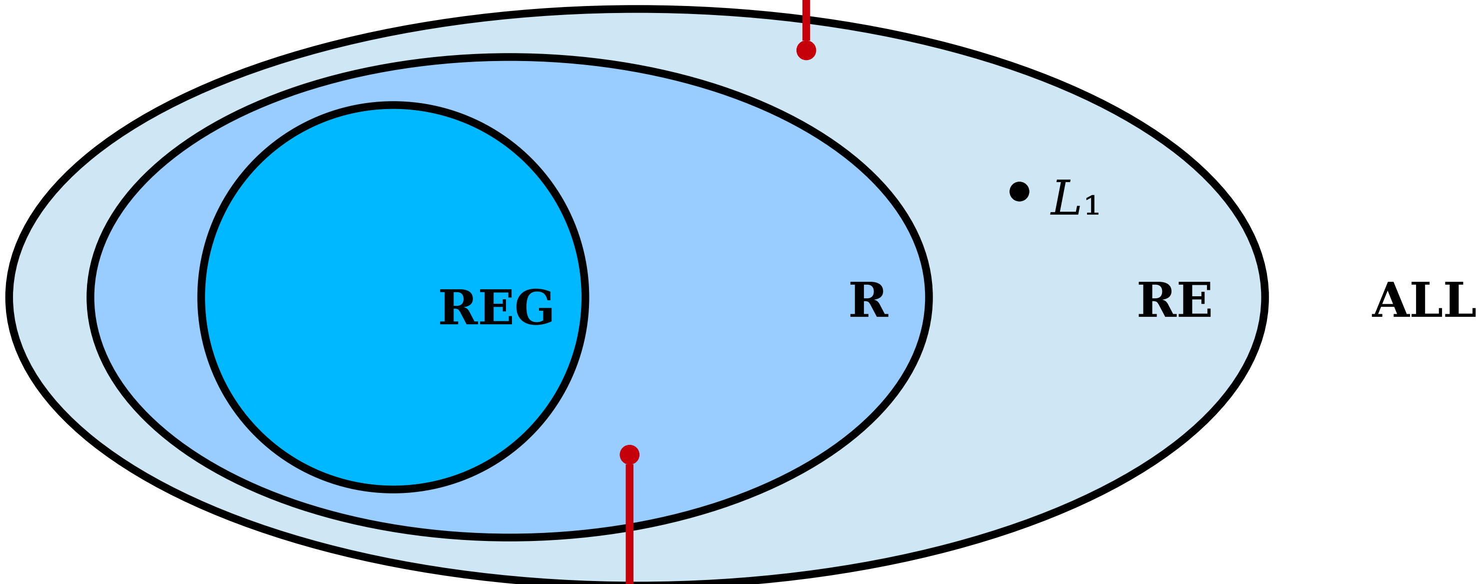
RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

A lot of people ask – "Isn't it really easy to build a TM that accepts at least two strings? So shouldn't this be decidable? Or even regular?"

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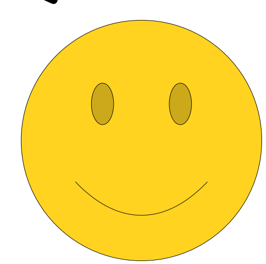
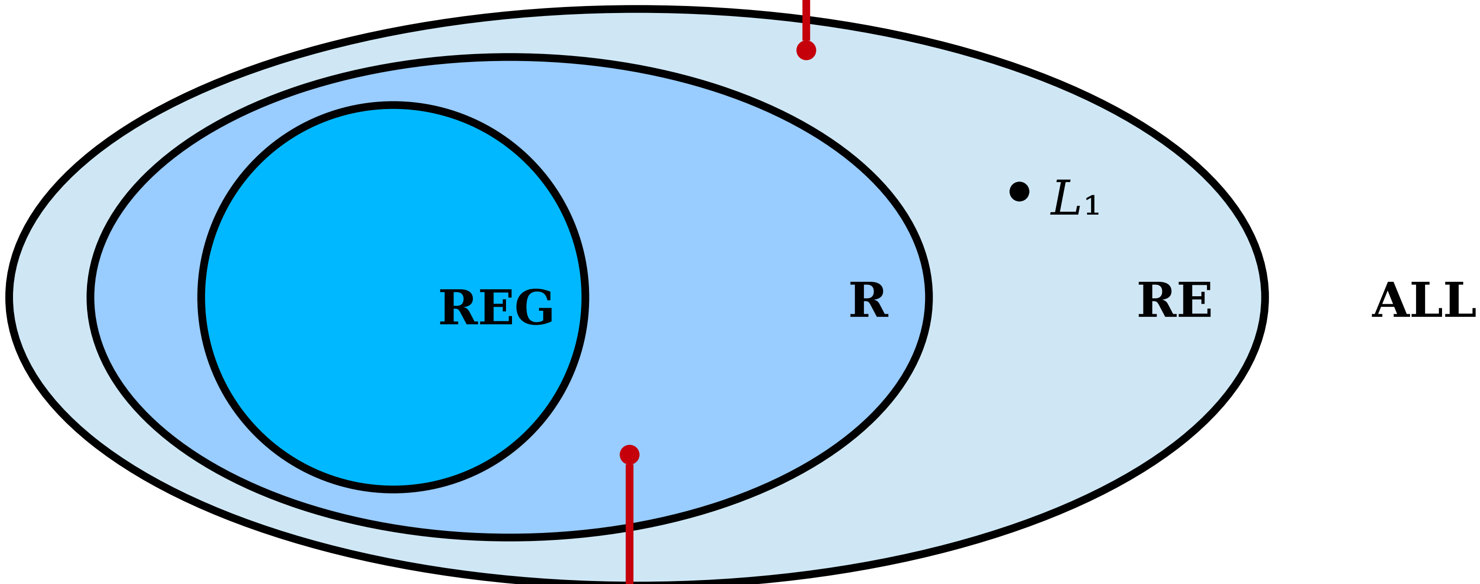


RE: Languages with Verifiers

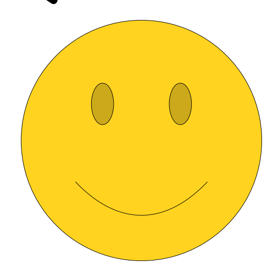
Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

The answer to that question is "yes, and no."



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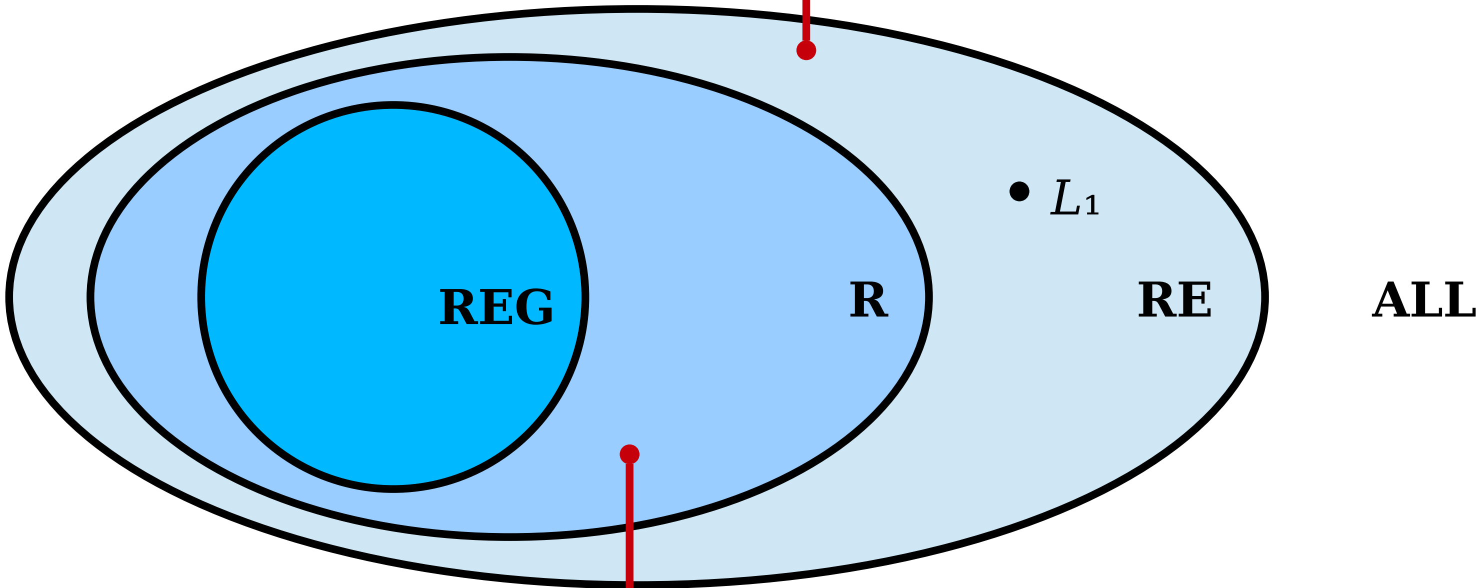
RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

L_1 = { ⟨M⟩ | M is a TM that accepts at least two strings }

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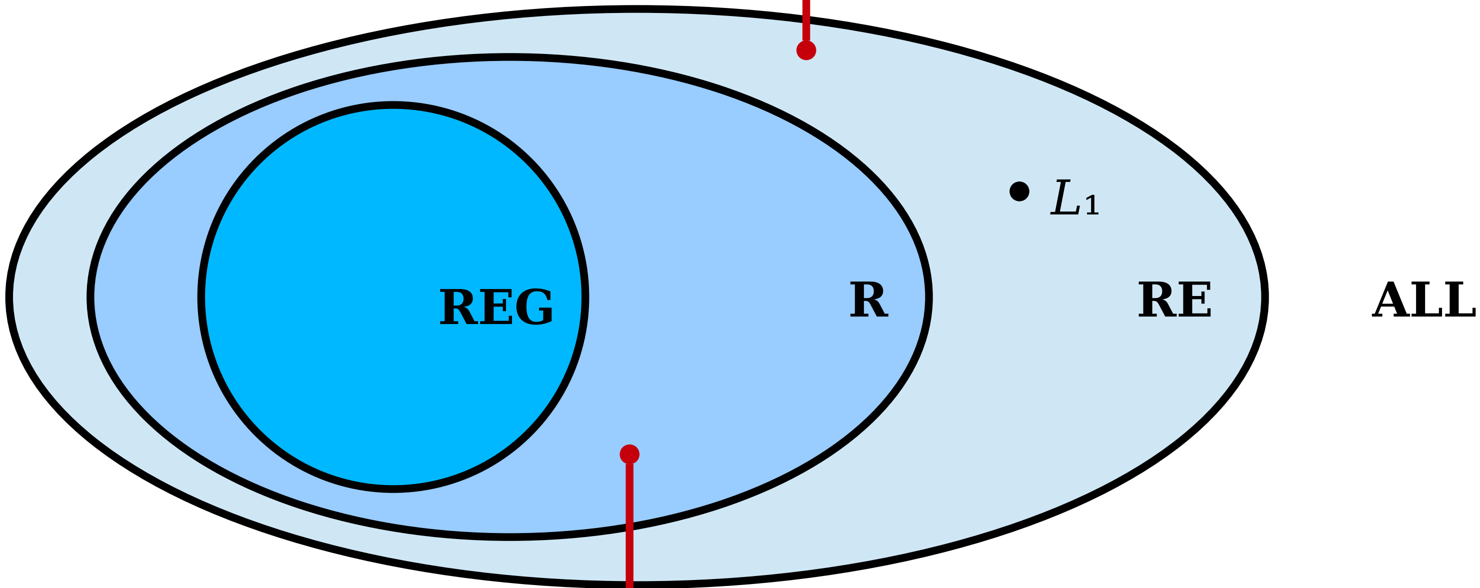
RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

We can do that by just building a TM that accepts everything, for example.

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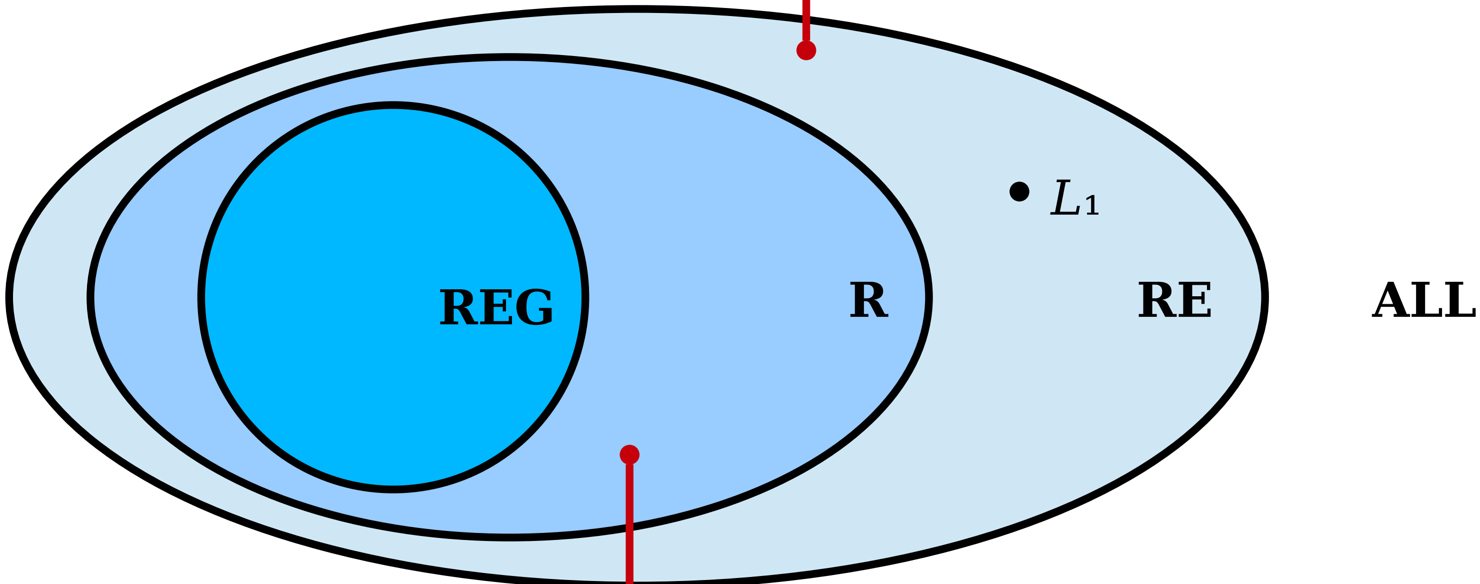
RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

But notice that this problem isn't asking whether you can build this machine. It's a question about the language of all TMs with this particular property.

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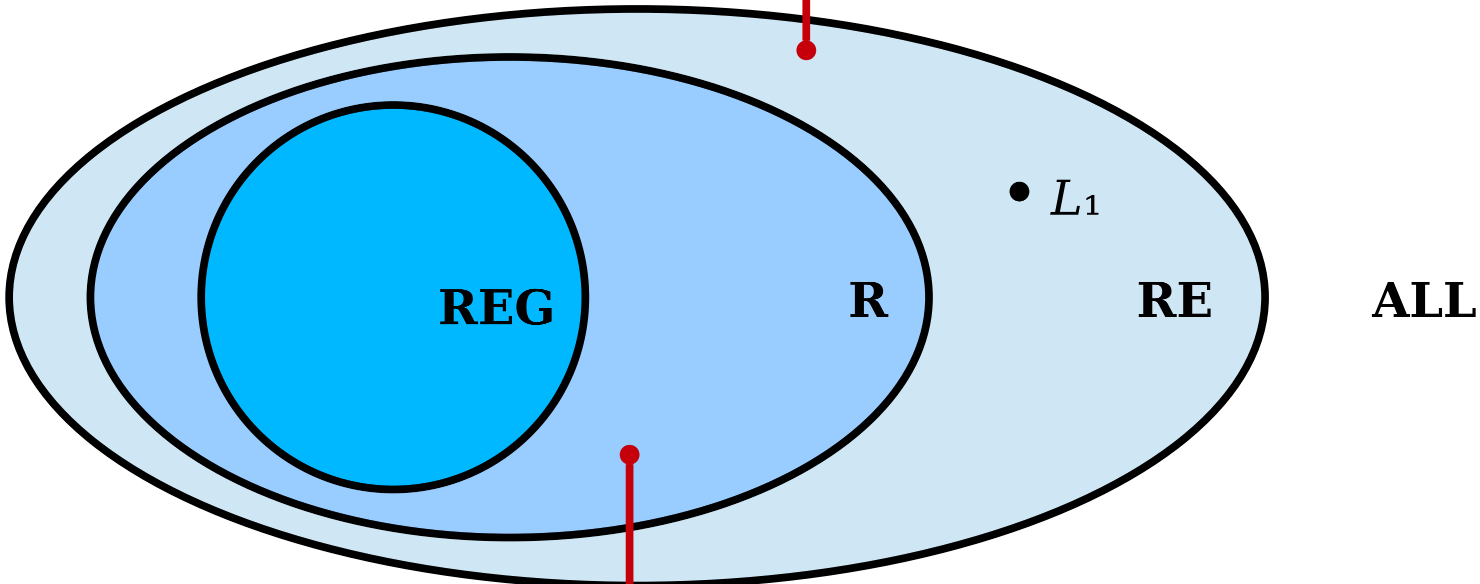
RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

In that sense, the question is really asking "how hard is it to tell whether a random TM actually does accept at least two strings?"

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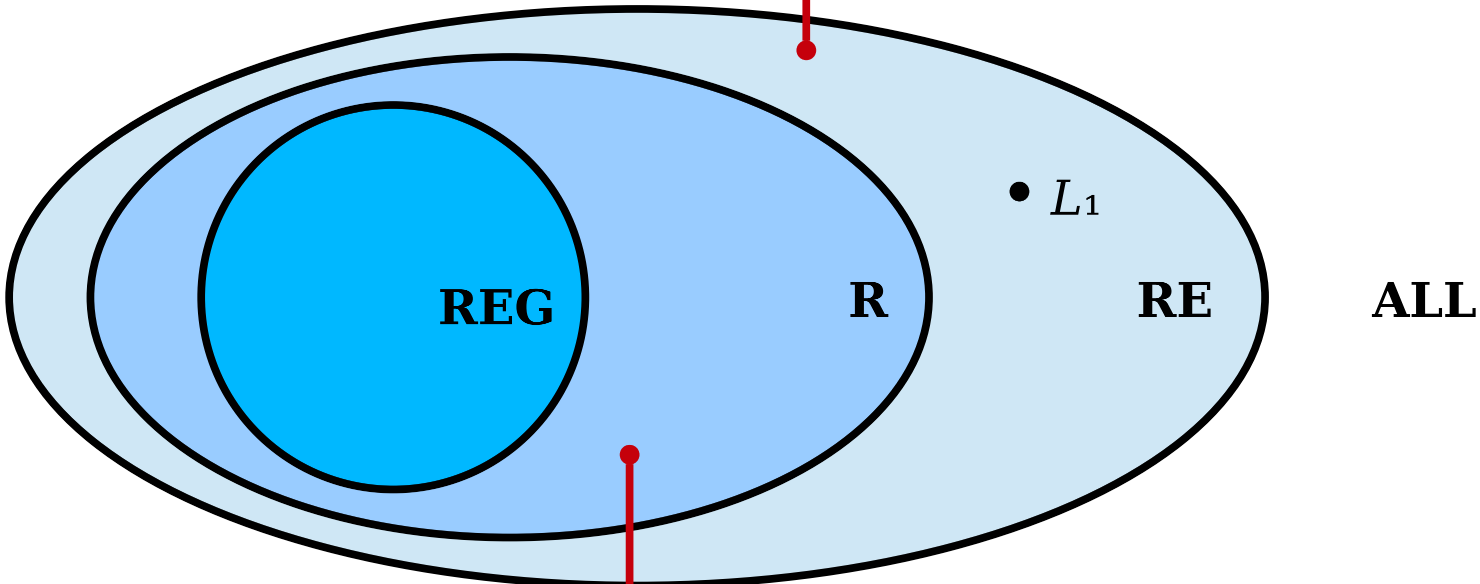
RE: Languages with Verifiers

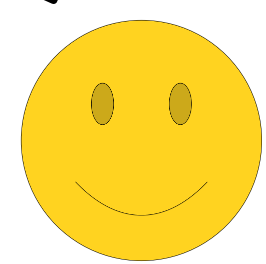
Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

That question – the question of checking whether a TM has some behavior – is typically much, much harder than the problem of building a TM with that behavior.



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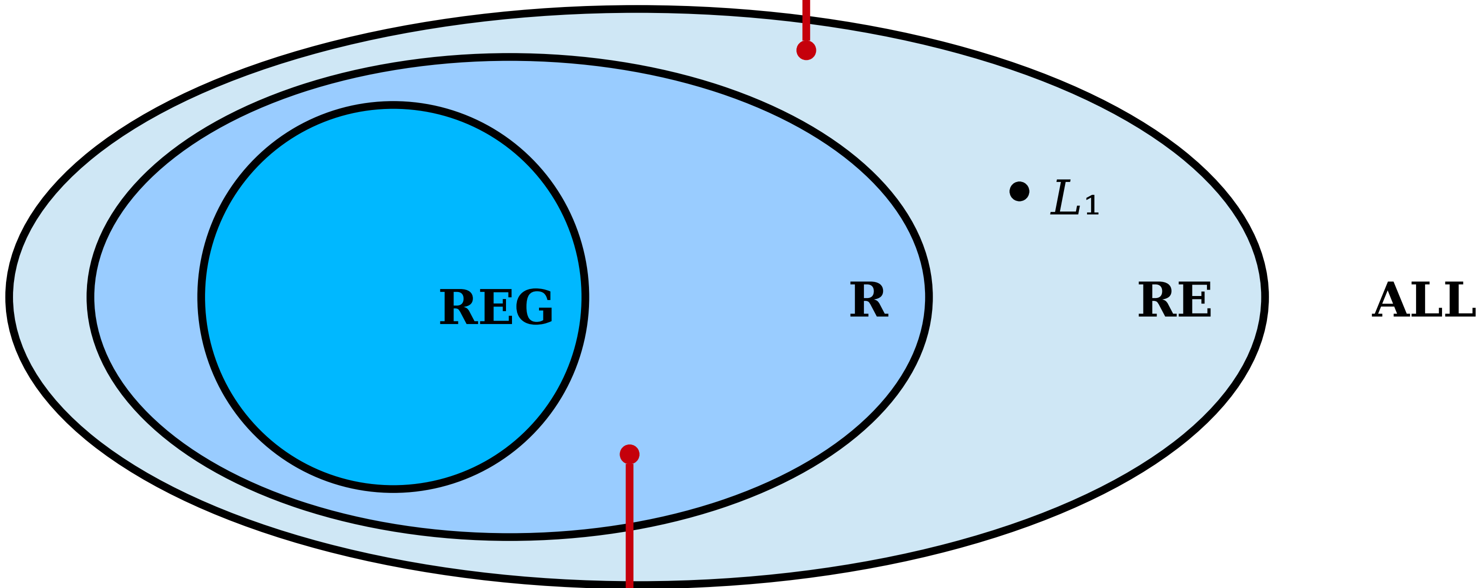
RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

Keep that in mind going forward – the question is to determine whether an arbitrary string is in the language, not to try to find a string that happens to be in the language.

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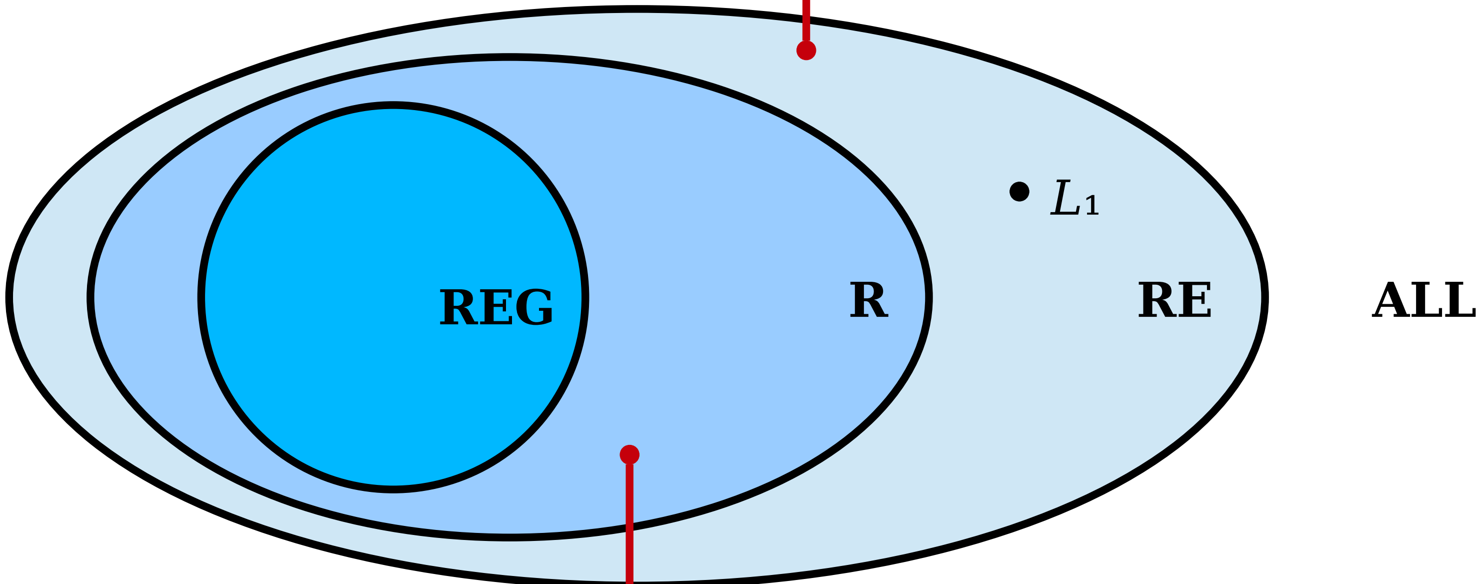
RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

And with that said, let's move on to the second language!

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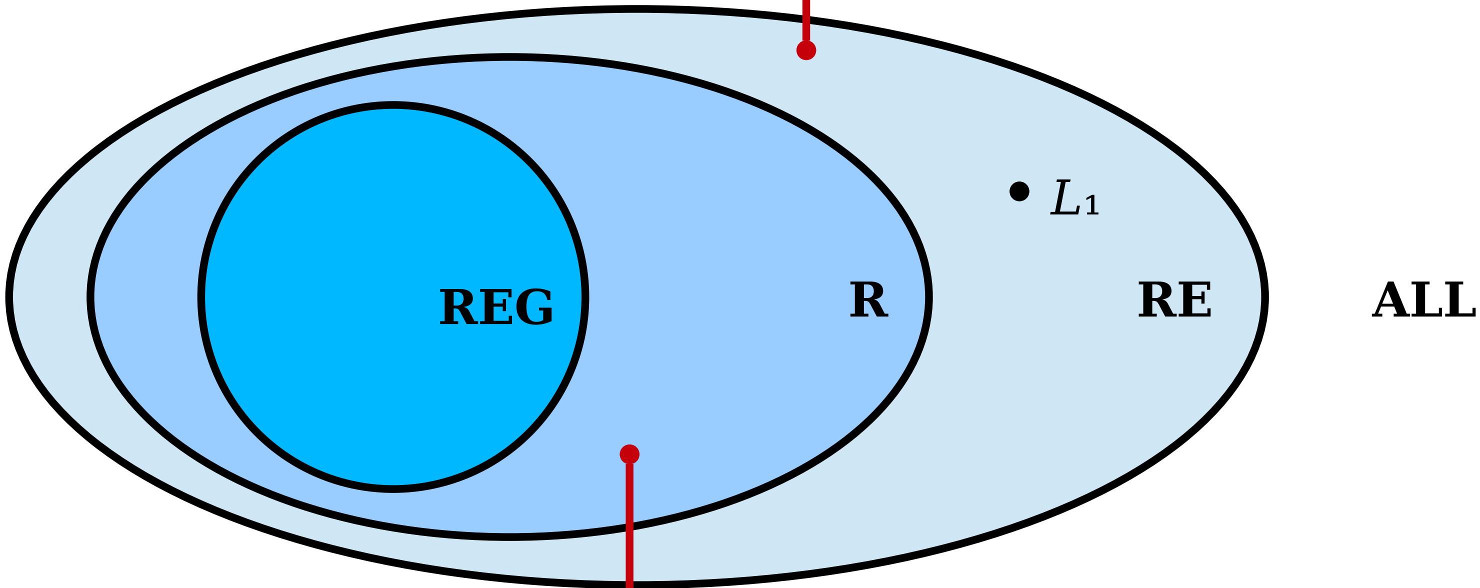
RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

Before I talk about this particular problem, take a few minutes to think about where you believe this should go in the Lava Diagram. Once you've done that, let's rejoin and keep talking.

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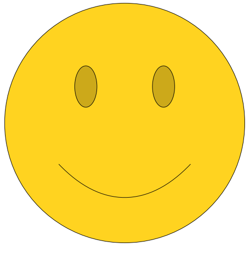
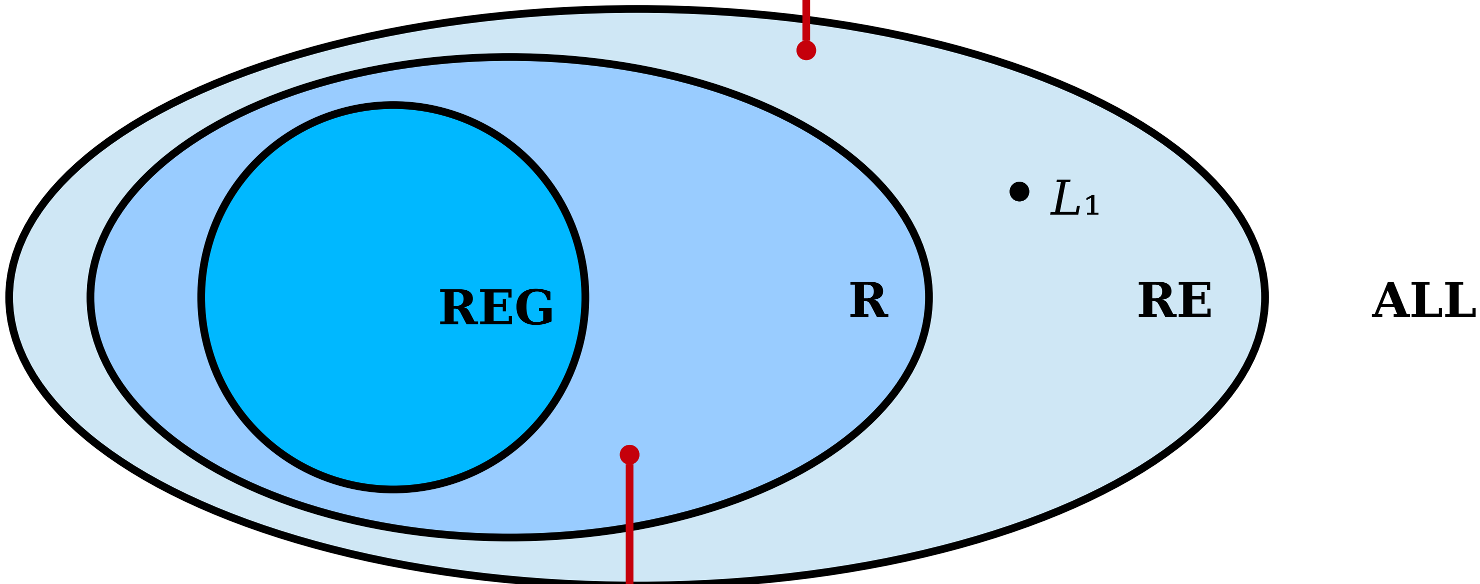


RE: Languages with Verifiers

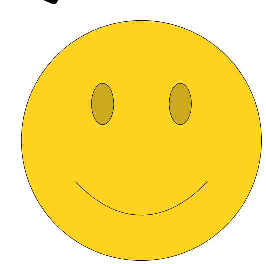
Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

Did you actually go and think about it? If not, you should. Like, seriously. It's good practice.



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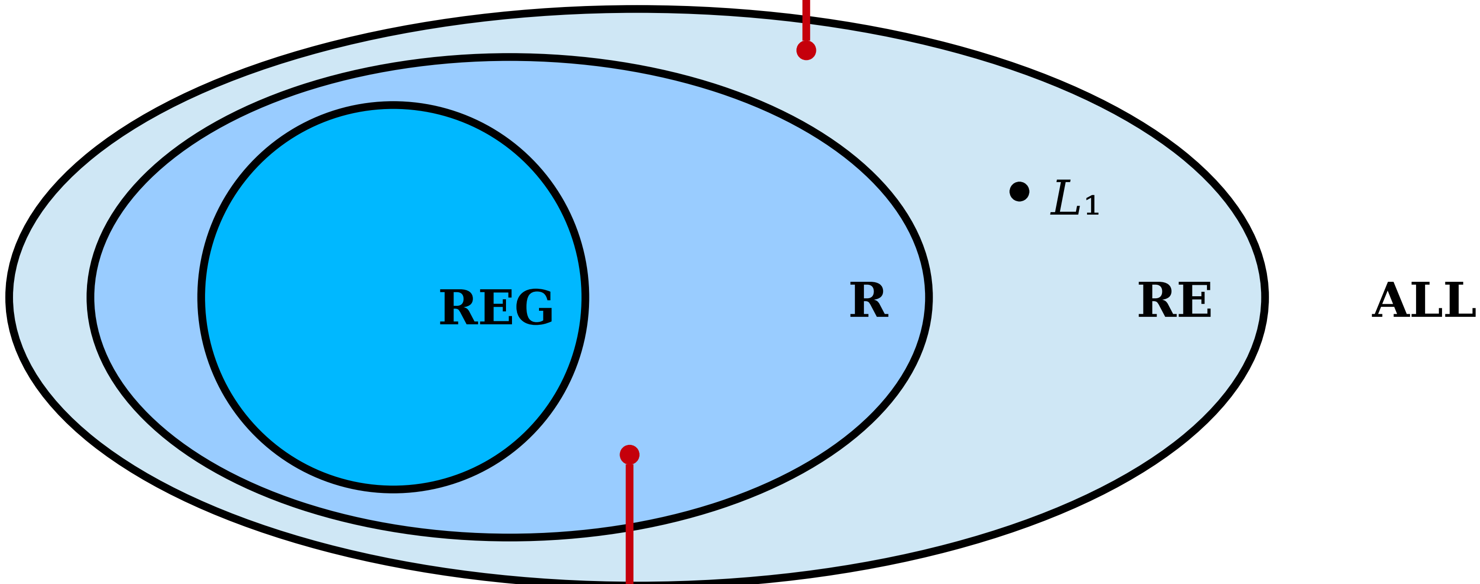
RE: Languages with Verifiers

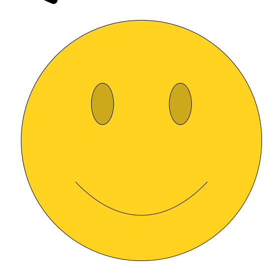
Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

Okay! So now you've given it your best shot. Let's see where this one goes.



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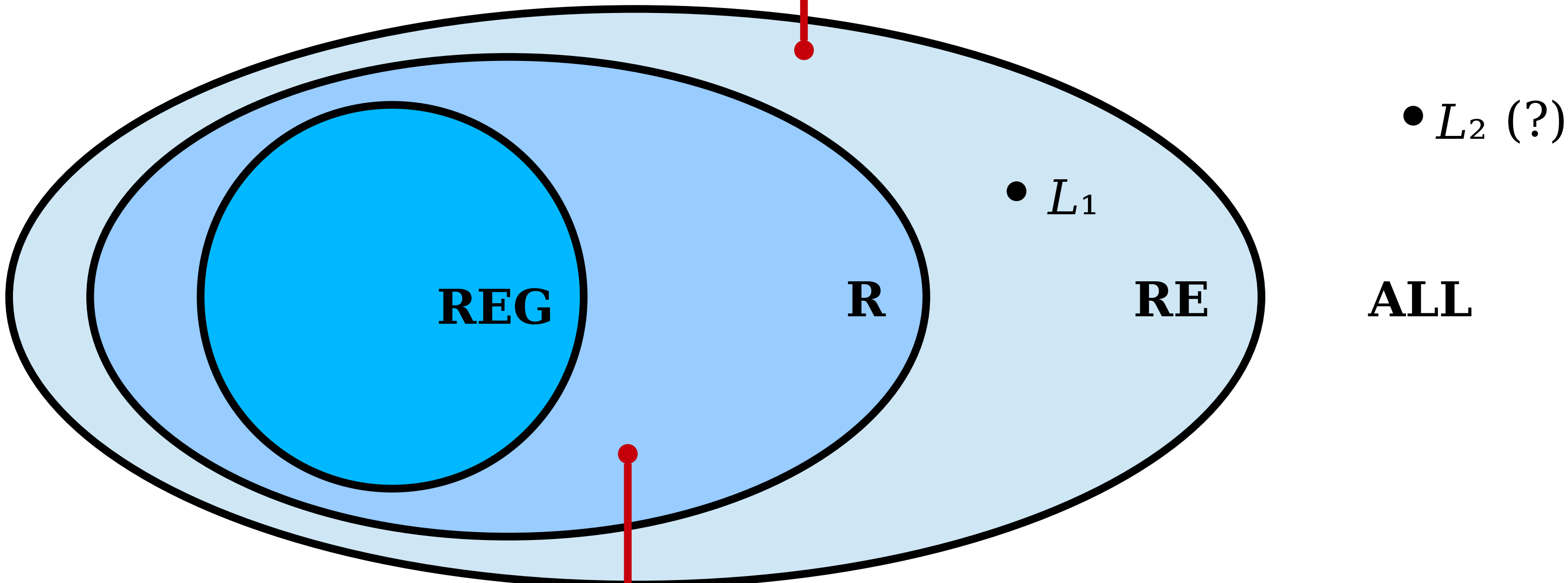
RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

As before, we're going to start on the outside and move inward. Initially, we won't make any assumptions about where this particular language goes.

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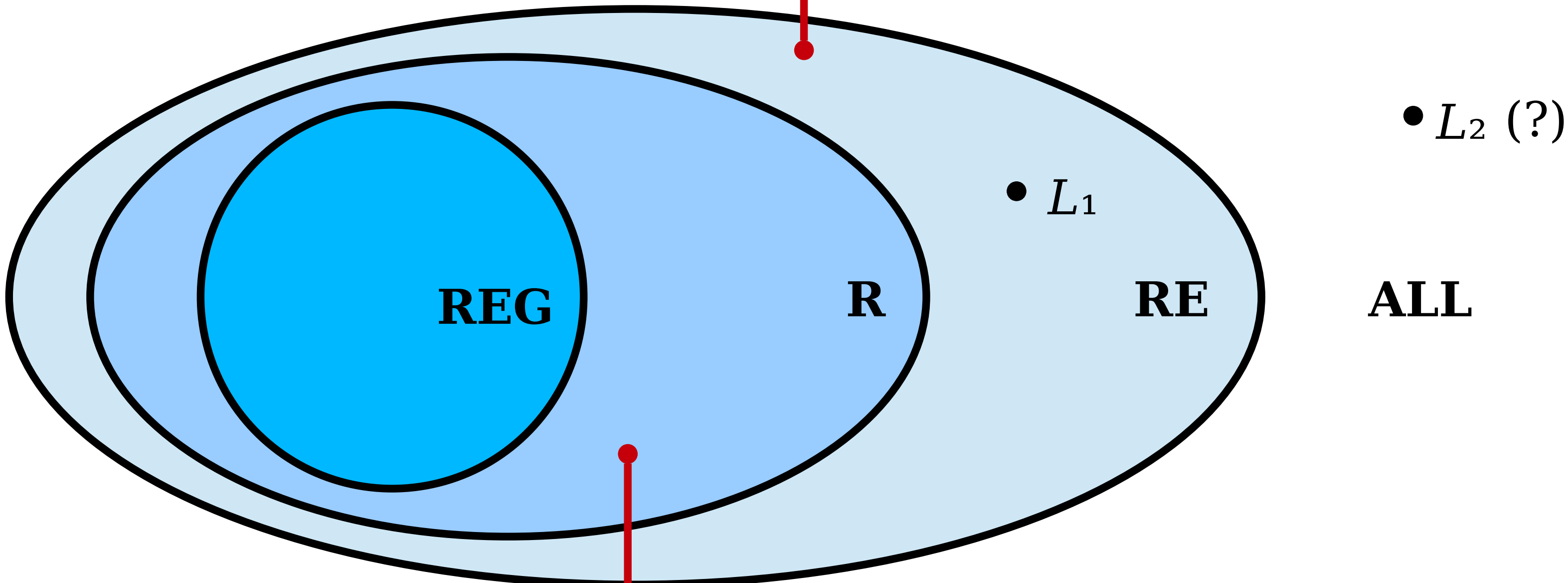
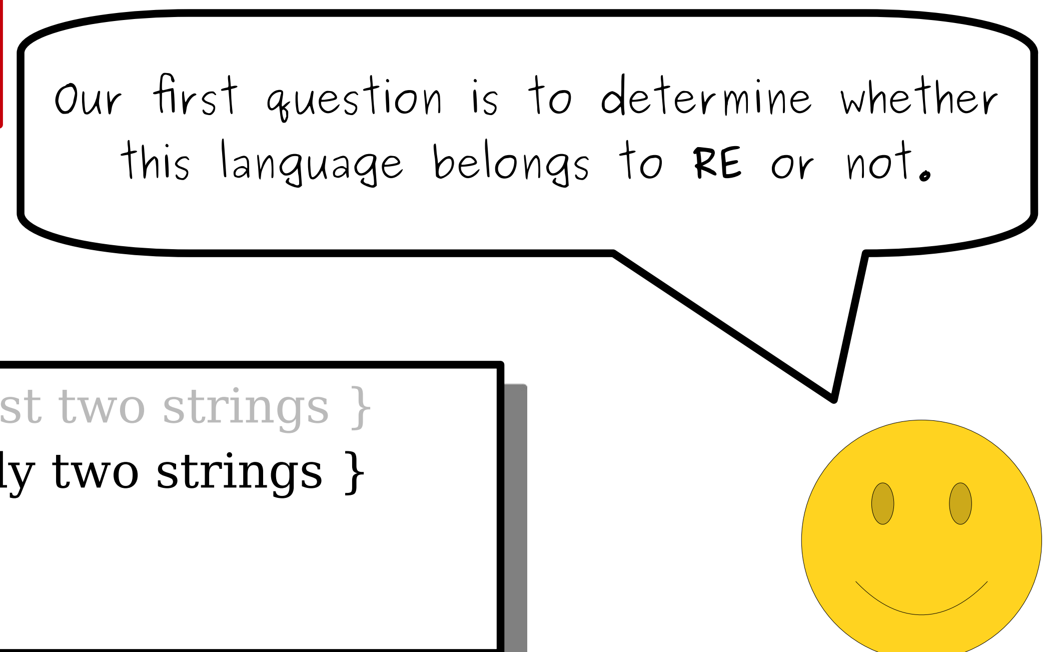
RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?



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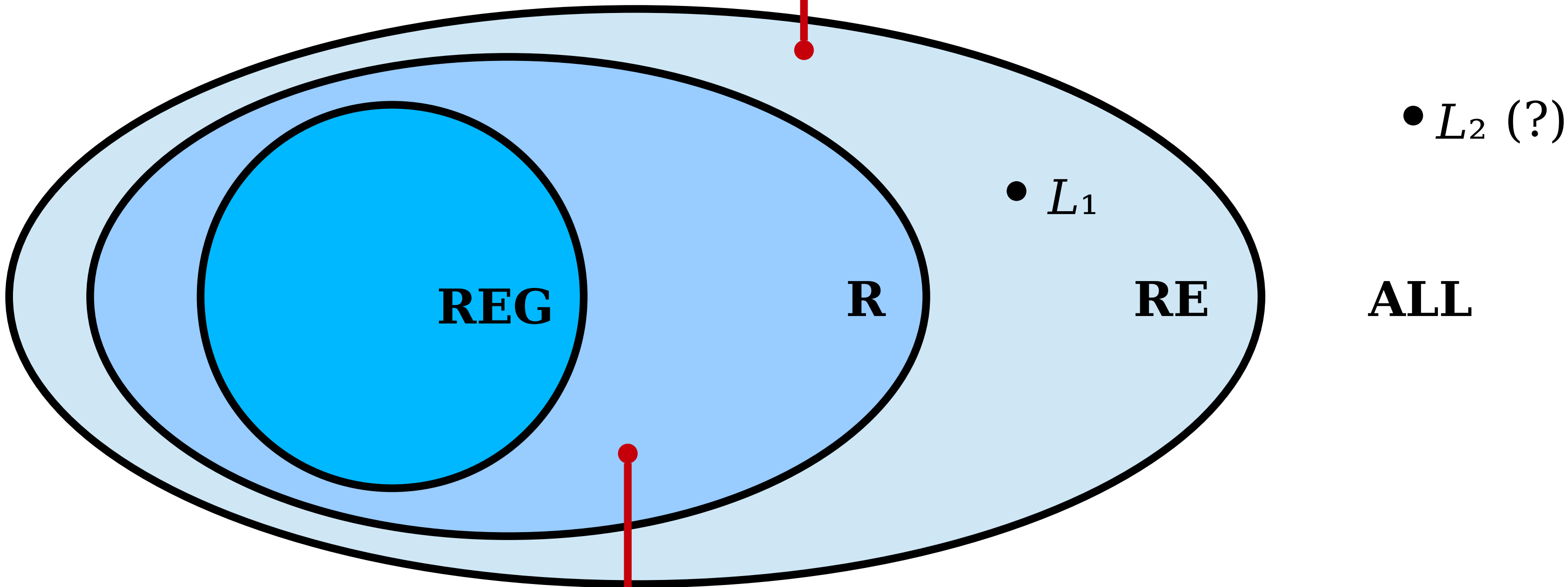
RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

To do so, we're going to ask whether, given a random string in the language, it's possible to prove it's in the language.

$L_1 = \{ \langle M \rangle \mid M$ is a TM that accepts at least two strings $\}$

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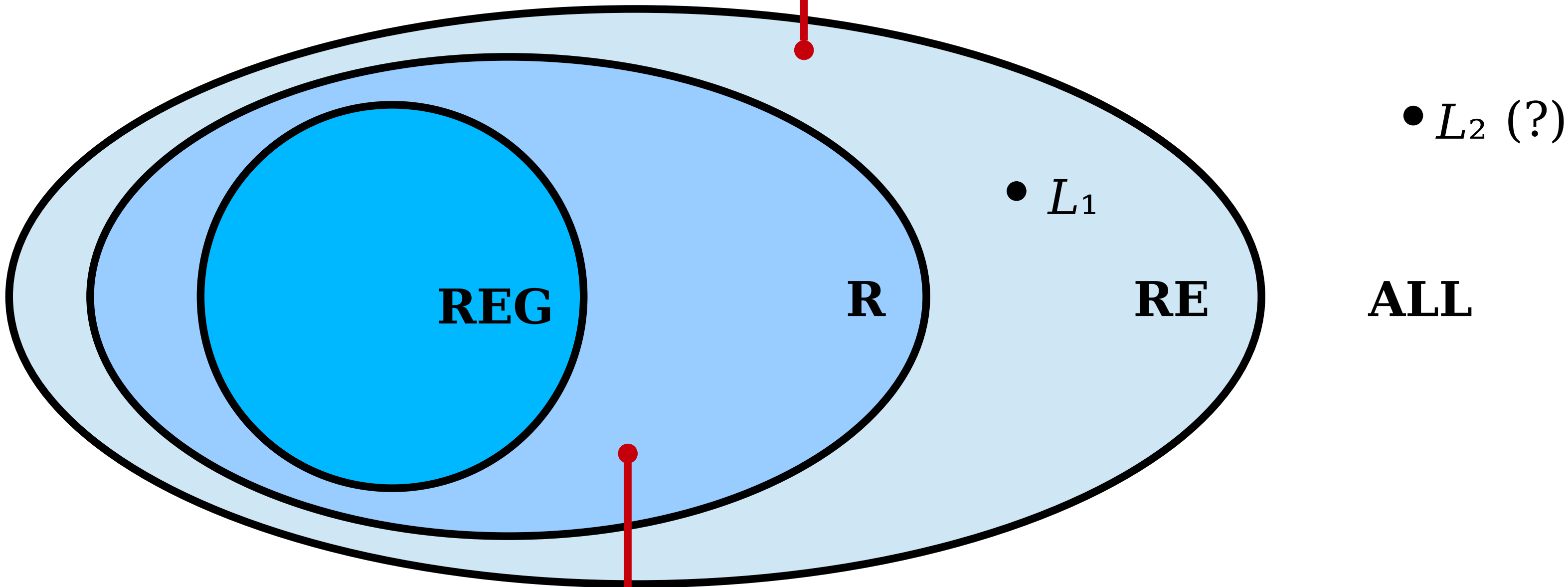
RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

This is the language of all TMs that accept exactly two strings.

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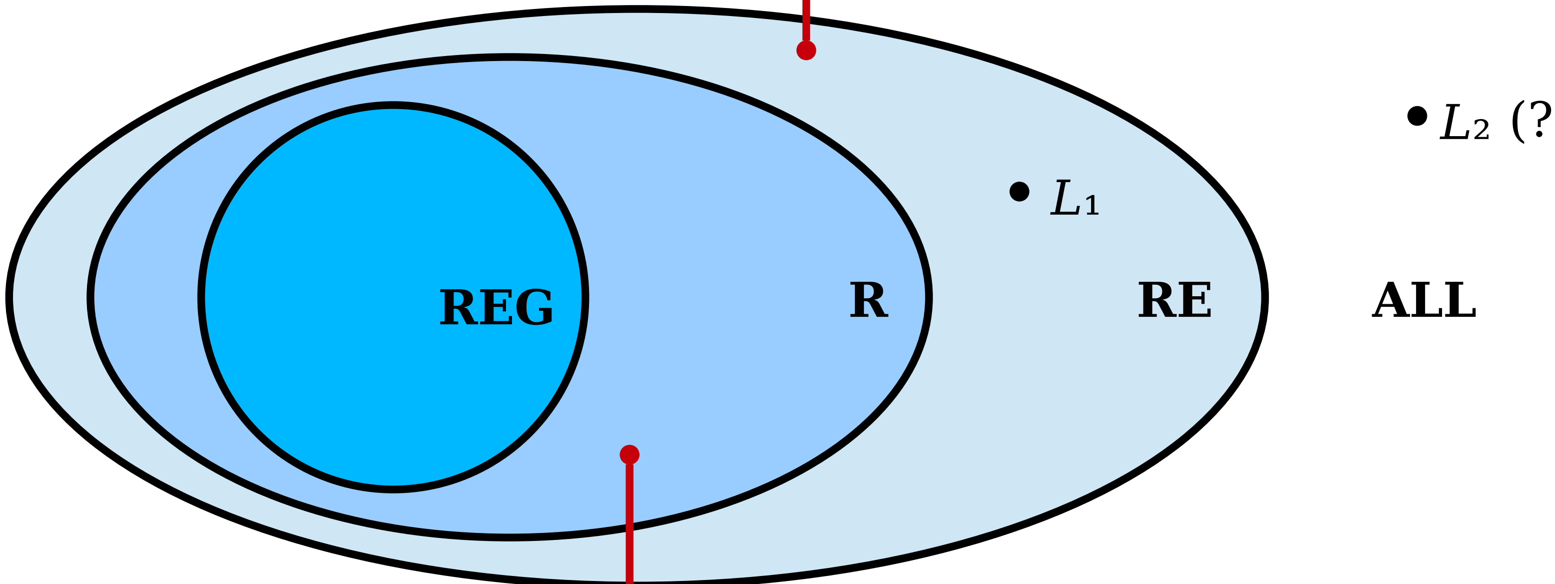
RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

So now we ask – if had a TM and you knew for a fact that it accepted exactly two strings, could you prove it?

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$L_3 = \{ \mathtt{a}^n\mathtt{b}^n \mid n \in \mathbb{N} \text{ and } n > 1000 \}$

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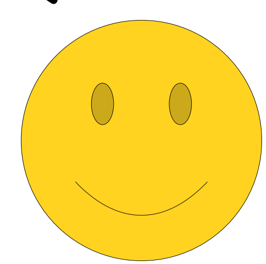
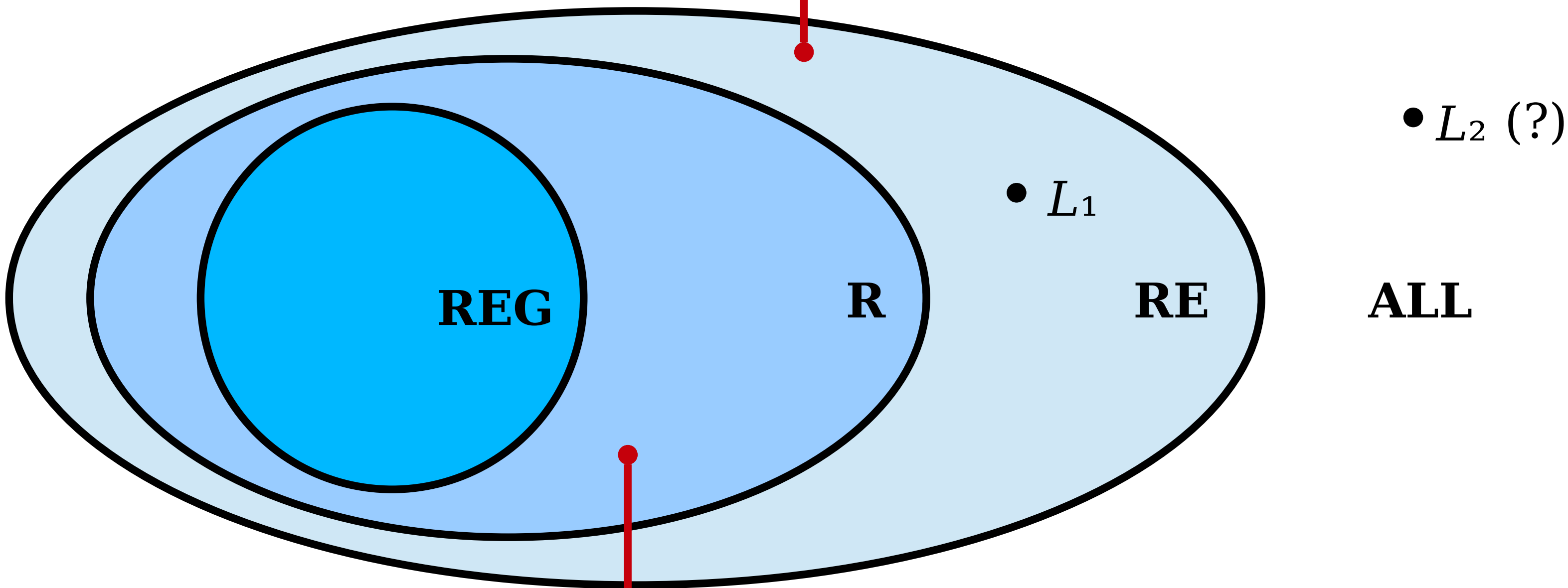


RE: Languages with Verifiers

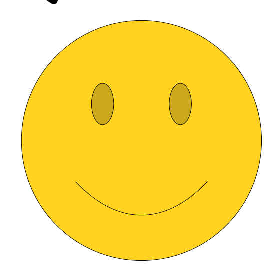
Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

This turns out to be a lot harder than just checking if a TM accepts at least two strings.



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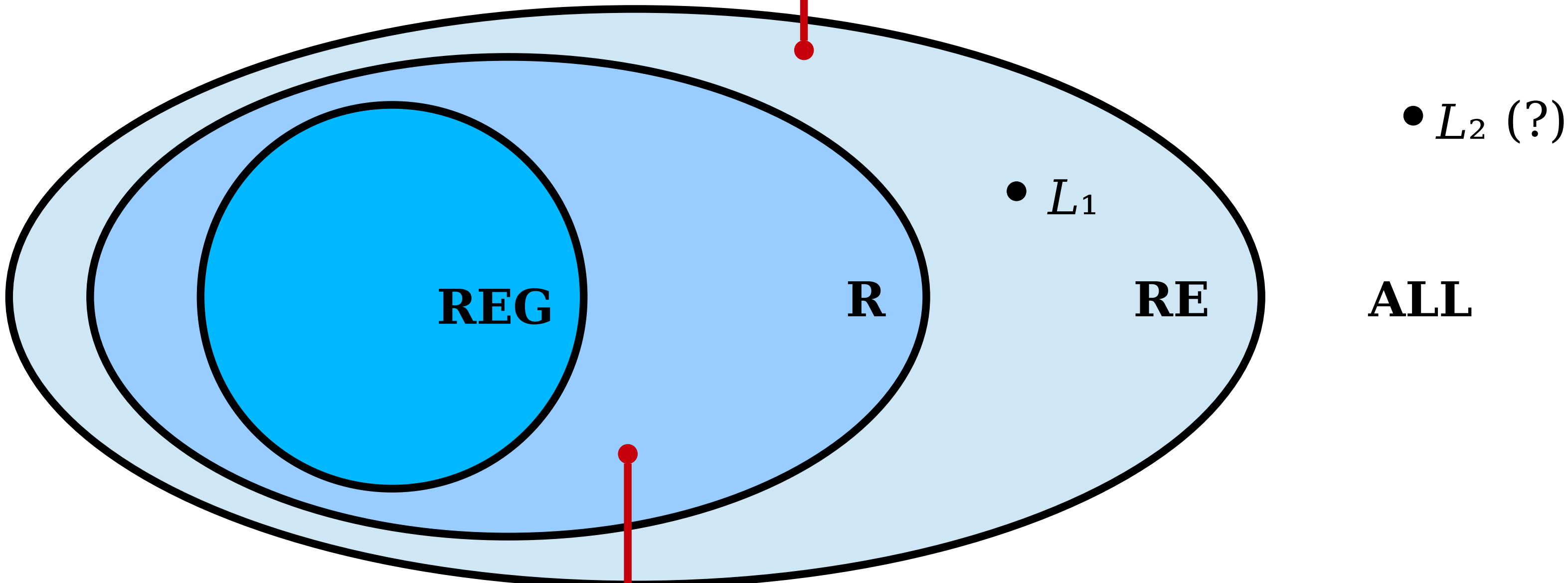
RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

To show that a TM accepts exactly two strings, we need to show that it accepts at least two strings (that's something we can prove), but also that it doesn't accept anything else.

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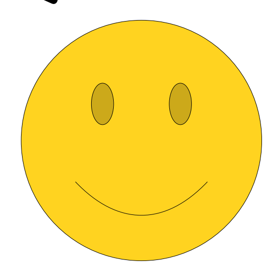
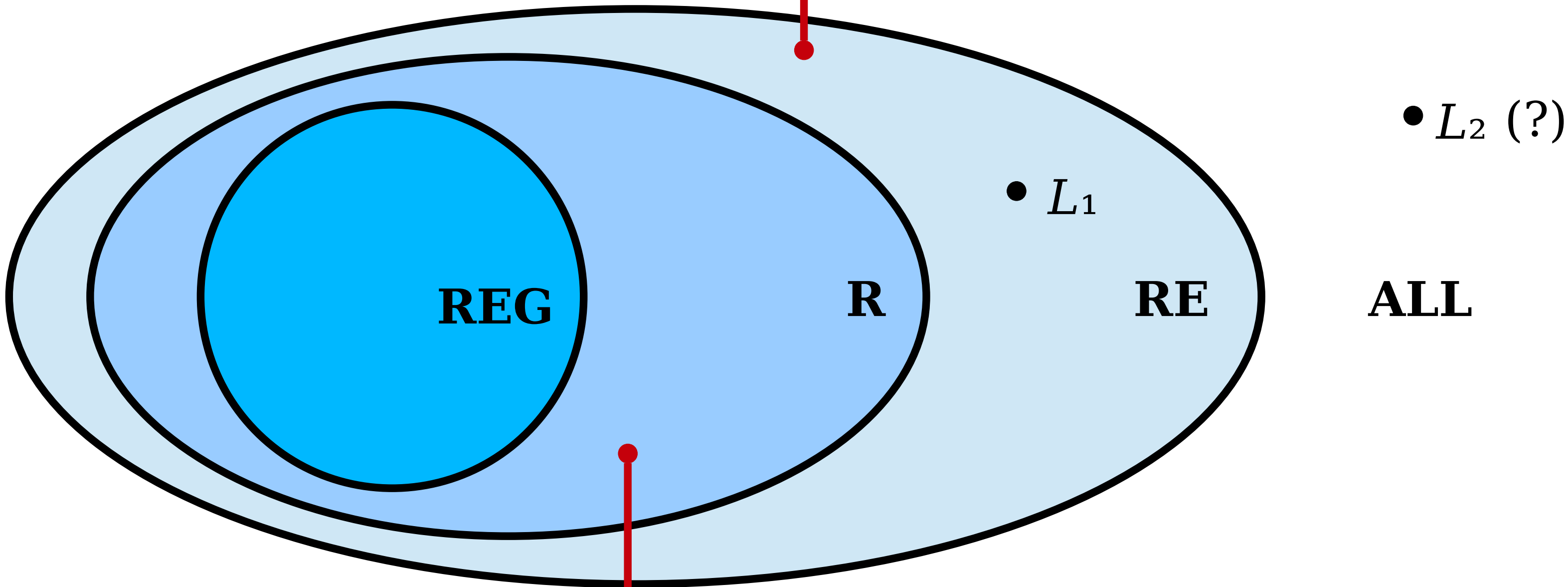


RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

The problem is that to show that a TM accepts a particular set of strings and nothing else, we need to prove that the TM doesn't accept any strings outside of that set.

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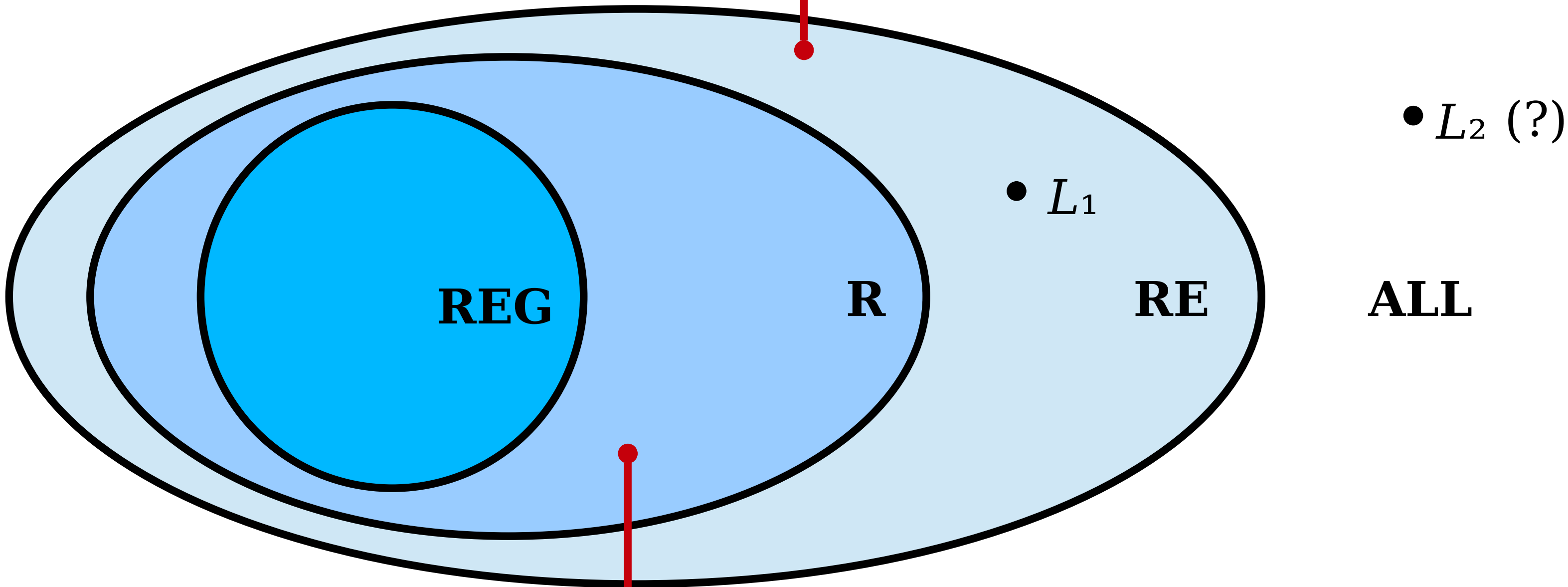

R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

That in turn would require us – in the general case – to run the TM on infinitely many strings to see what happens, since there's no general way to see what a TM does other than to run it.

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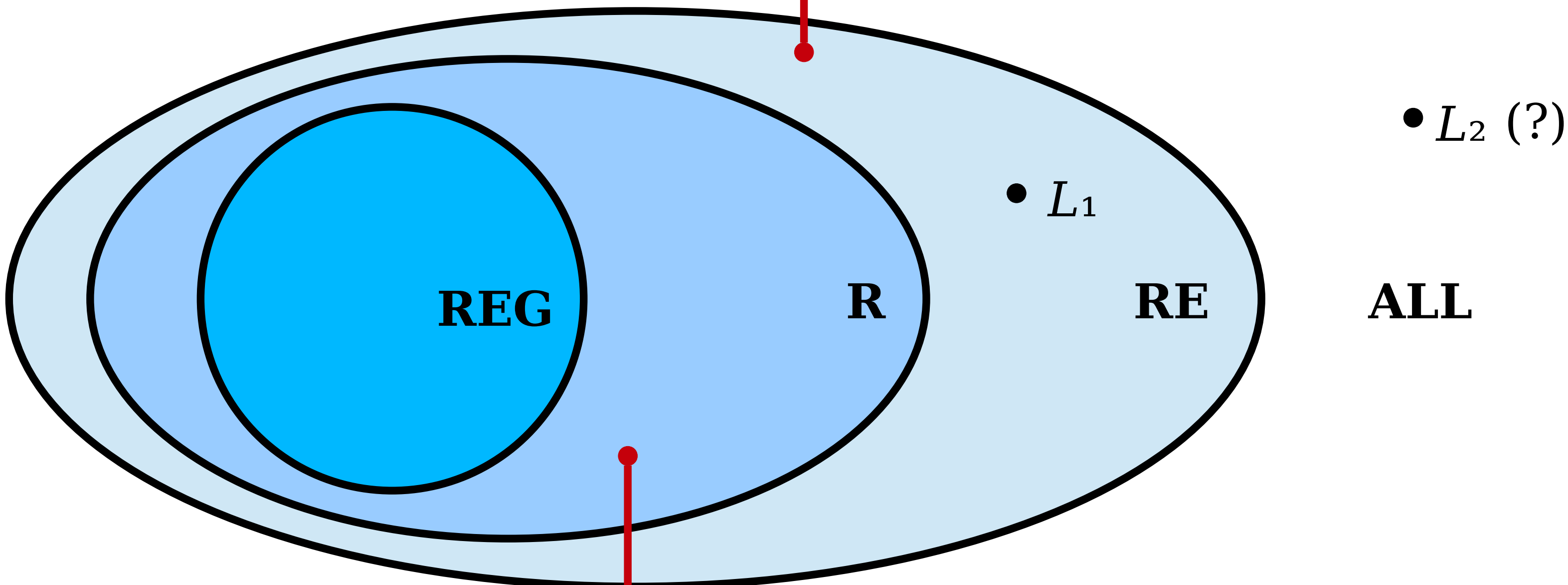

R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

So at least, intuitively, this doesn't seem like it's going to be possible to do. Even if we know that TM accepts exactly two strings, it's unclear how we'd prove that to someone.

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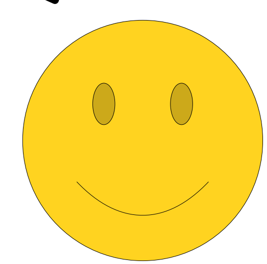

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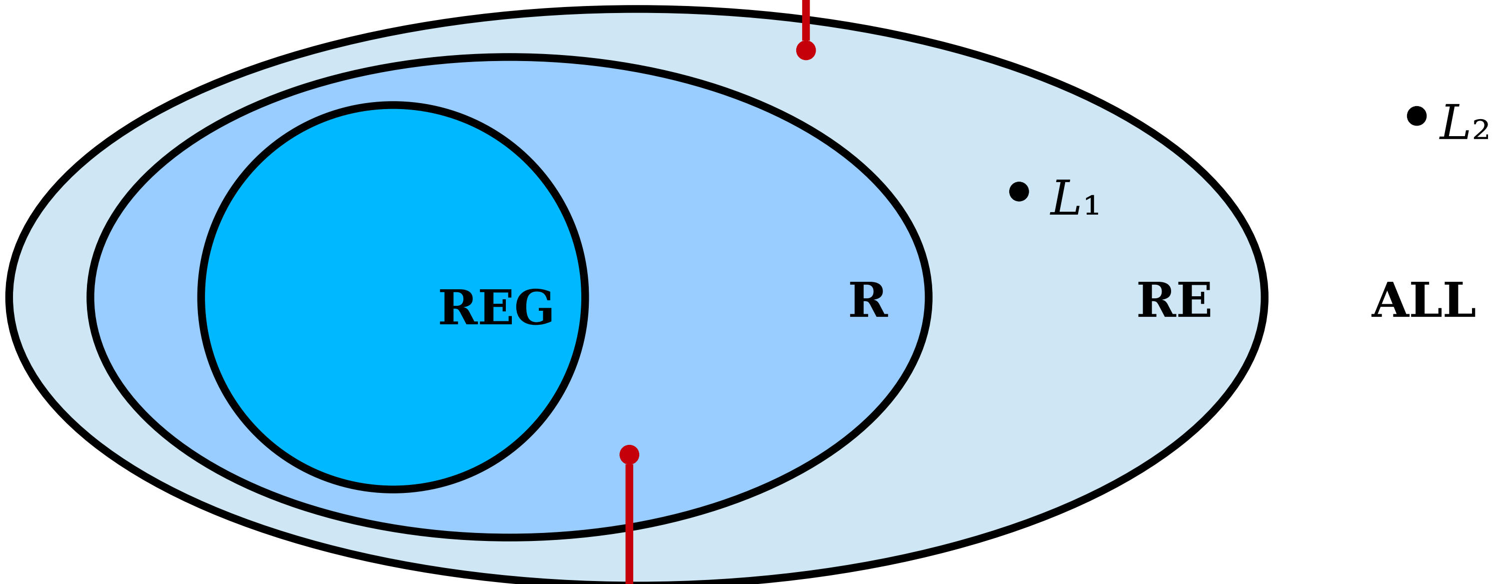

R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

This gives us some justification to guess that this language is probably not going to be in **RE**.

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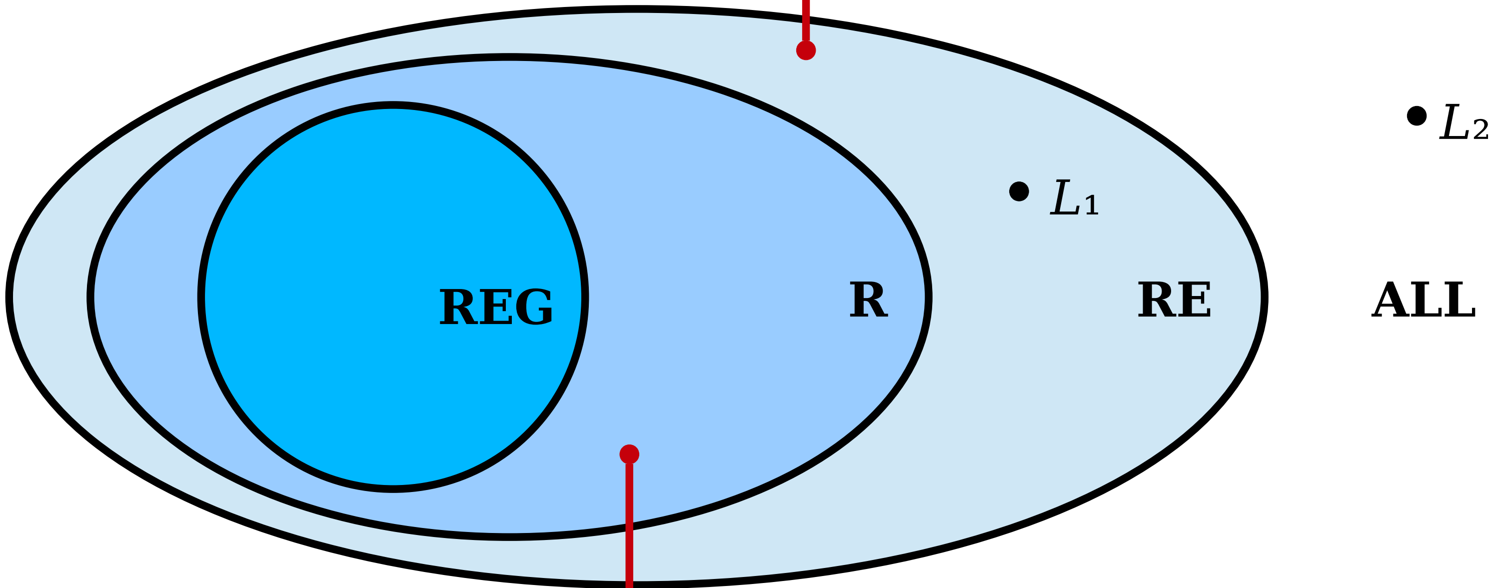

R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

So there you have it – this language is not even in RE.

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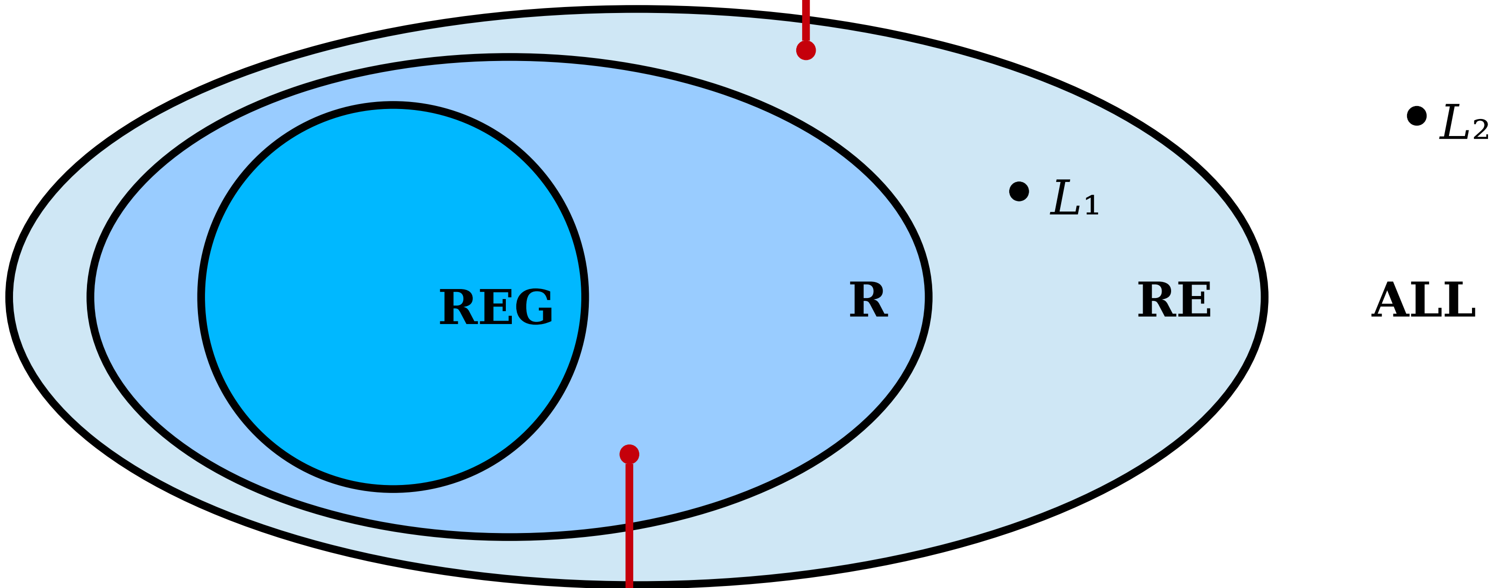

R: Languages with Deciders

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That might seem pretty surprising, given how similar this language looks to L_1.

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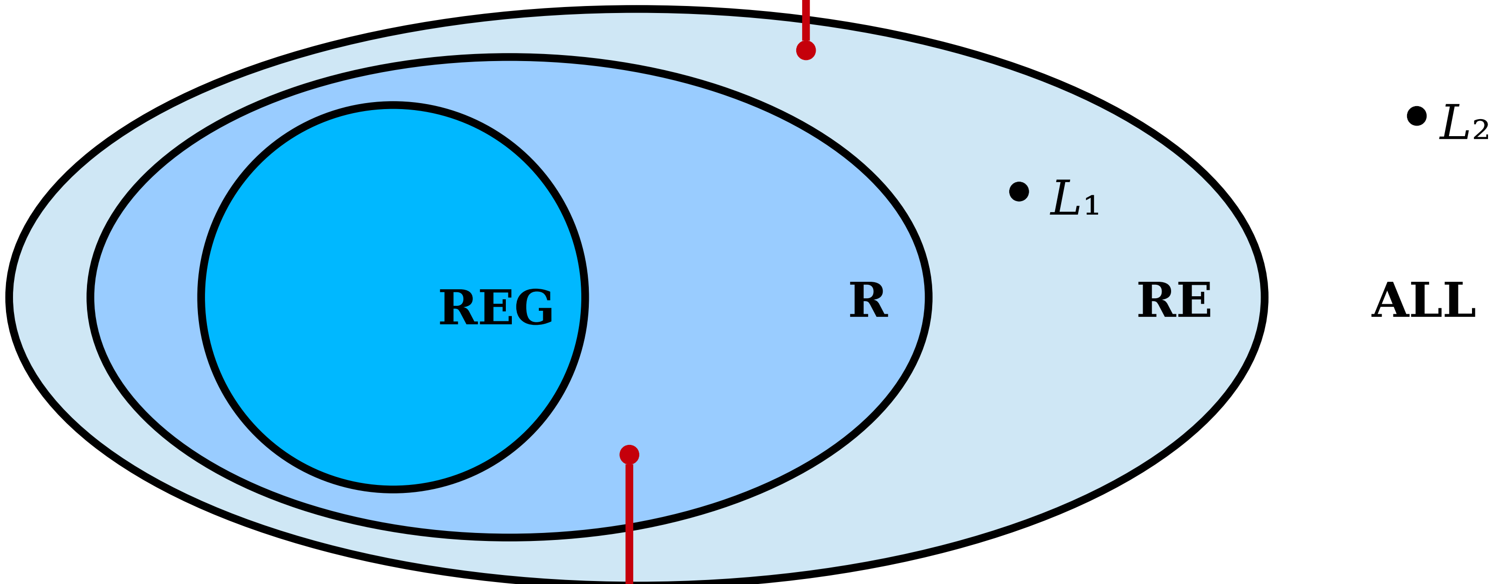

R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

I chose this particular example because it highlights a key point when thinking about languages: don't try to place a language in the diagram just based on its description.

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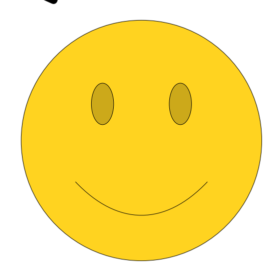

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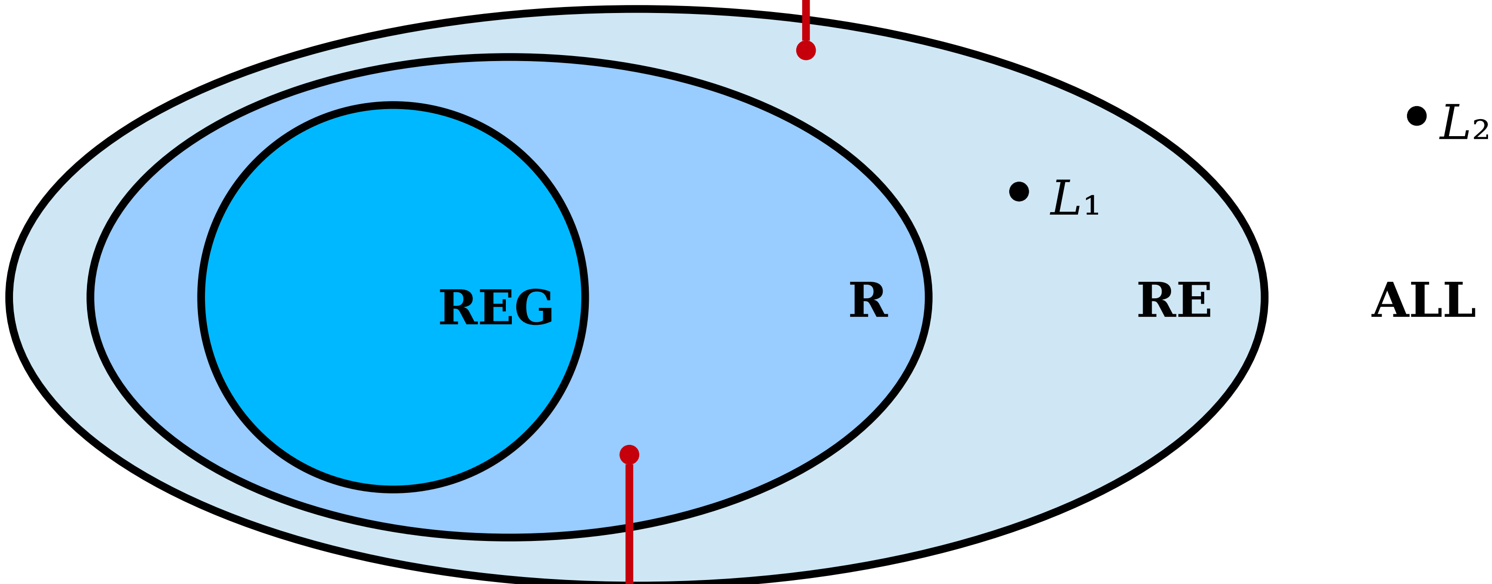

R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

To figure out where something goes, you need to think about in terms of provability. Ultimately, it's this – rather than the way it's written – that makes things hard.

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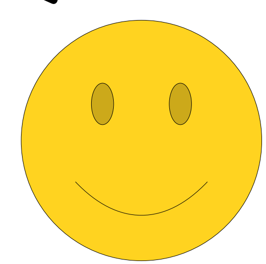

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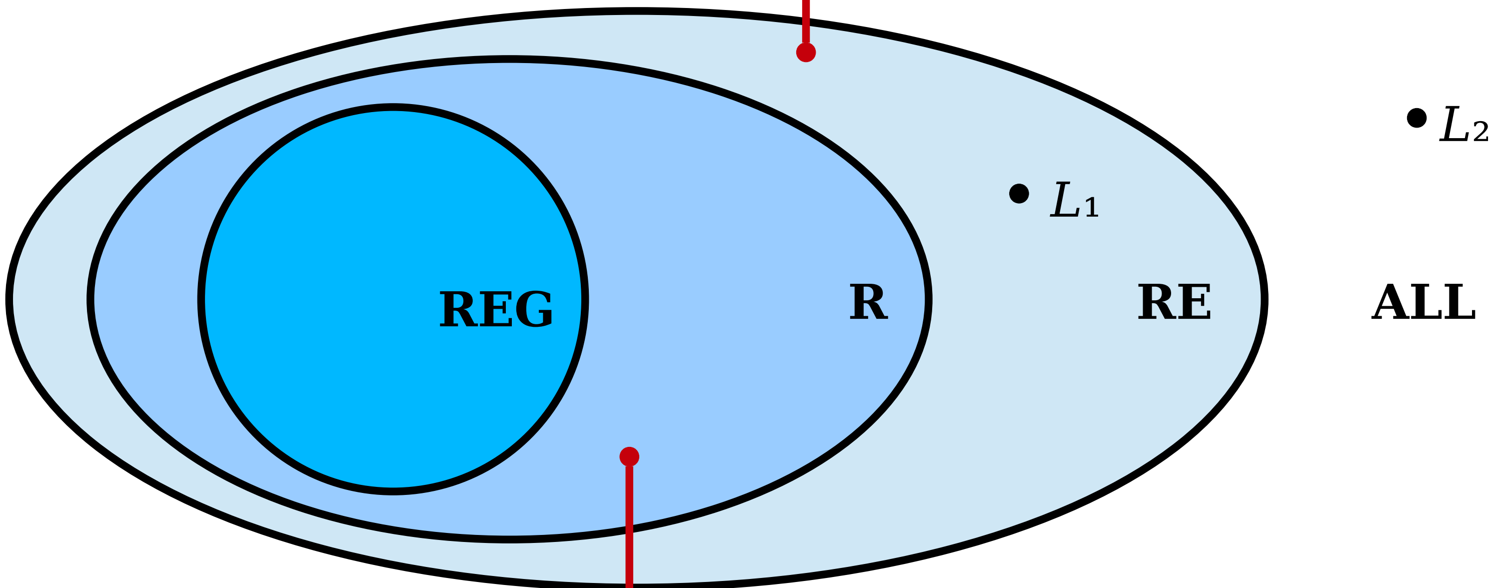

R: Languages with Deciders

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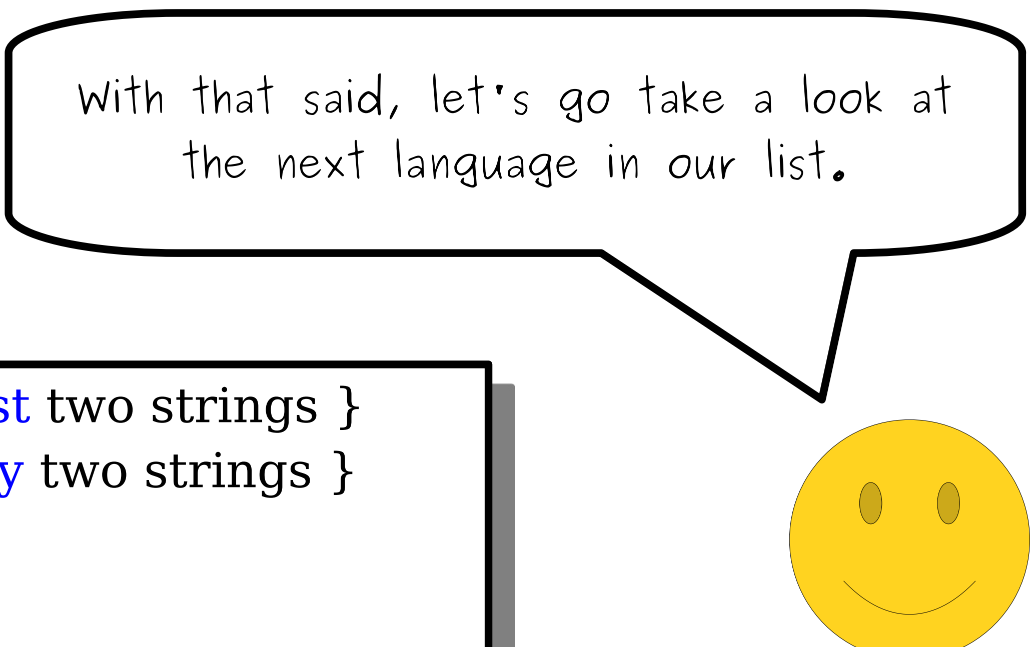

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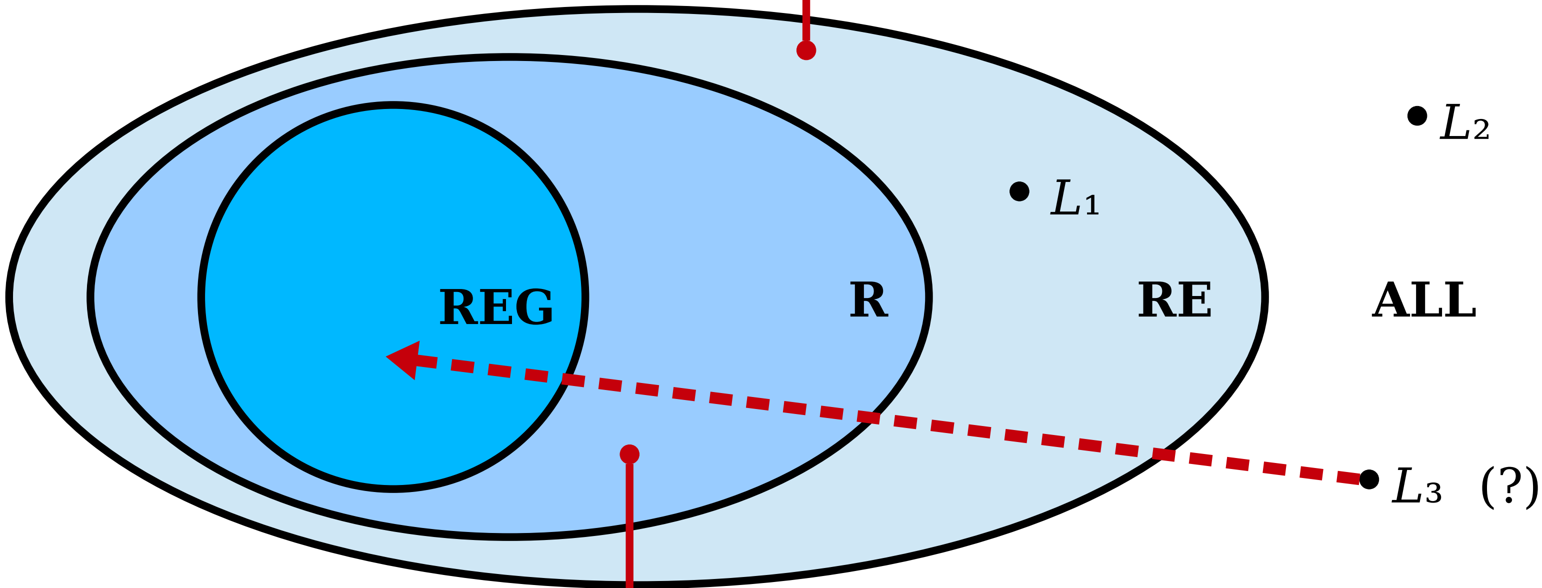

R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

As before, we'll start by placing it outside of **RE** and try to think about pushing it as far down as possible.

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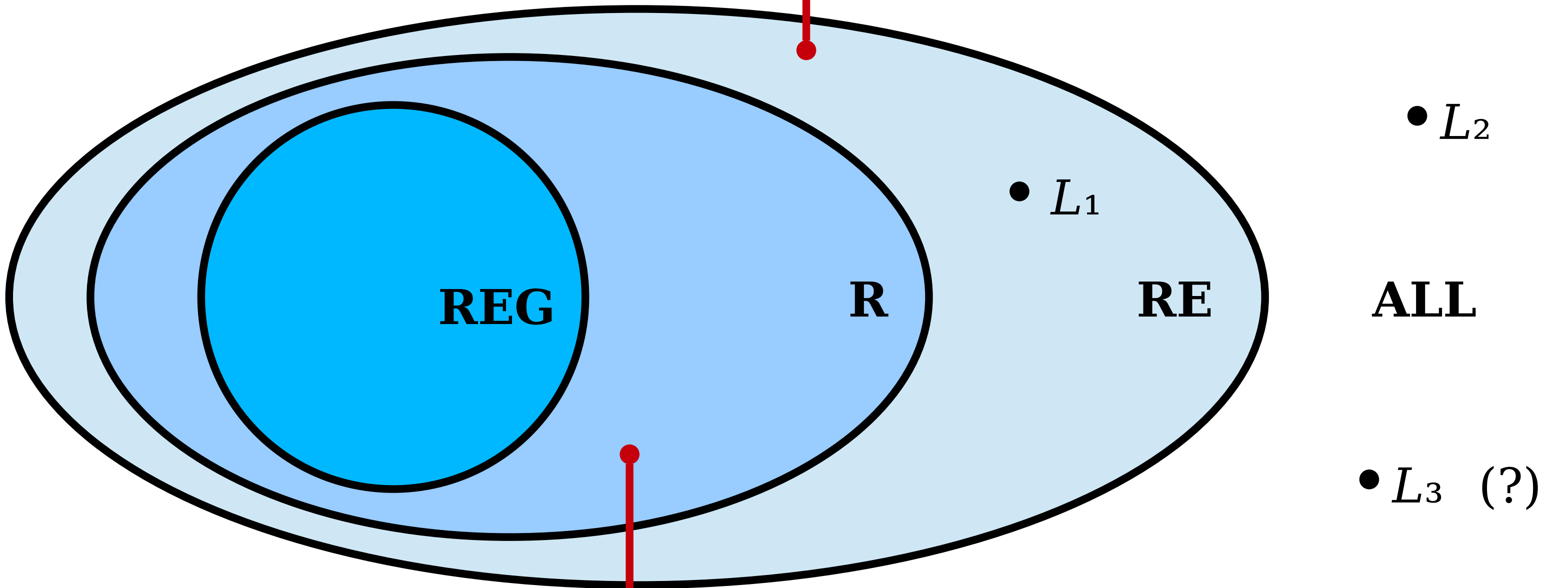

R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

As before, we first ask whether this language happens to be in **RE**.

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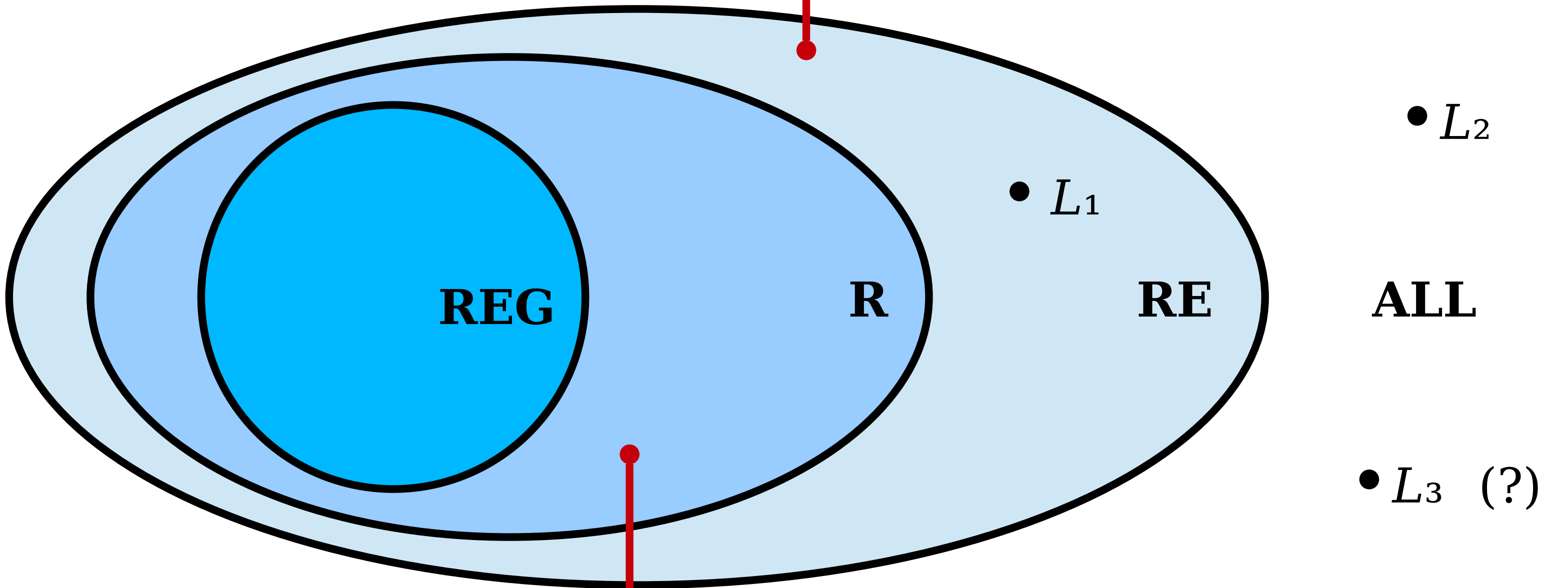

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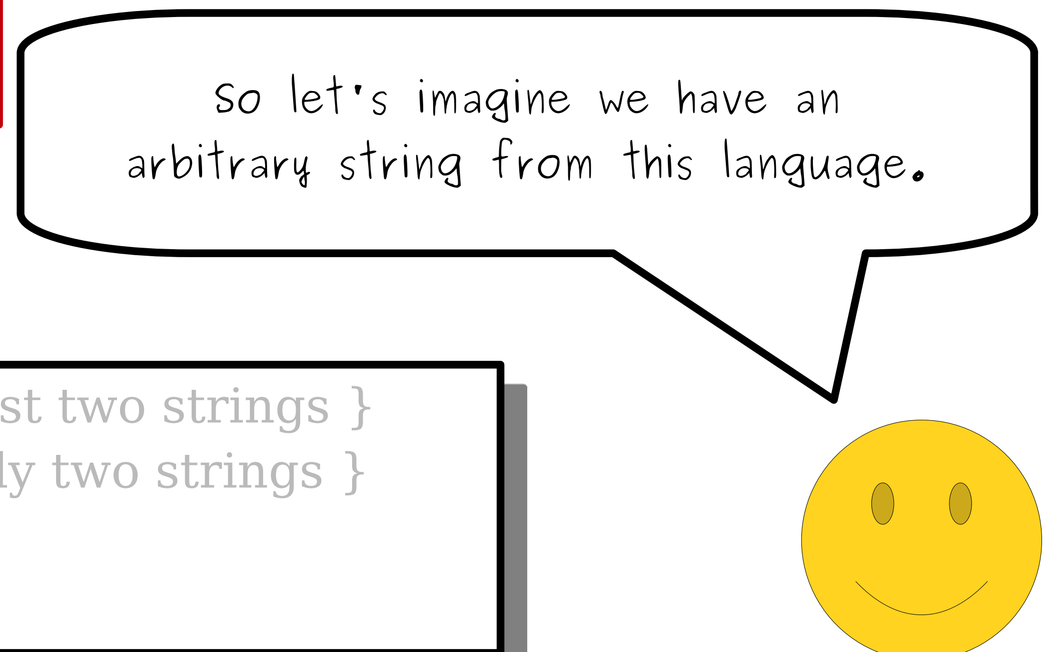

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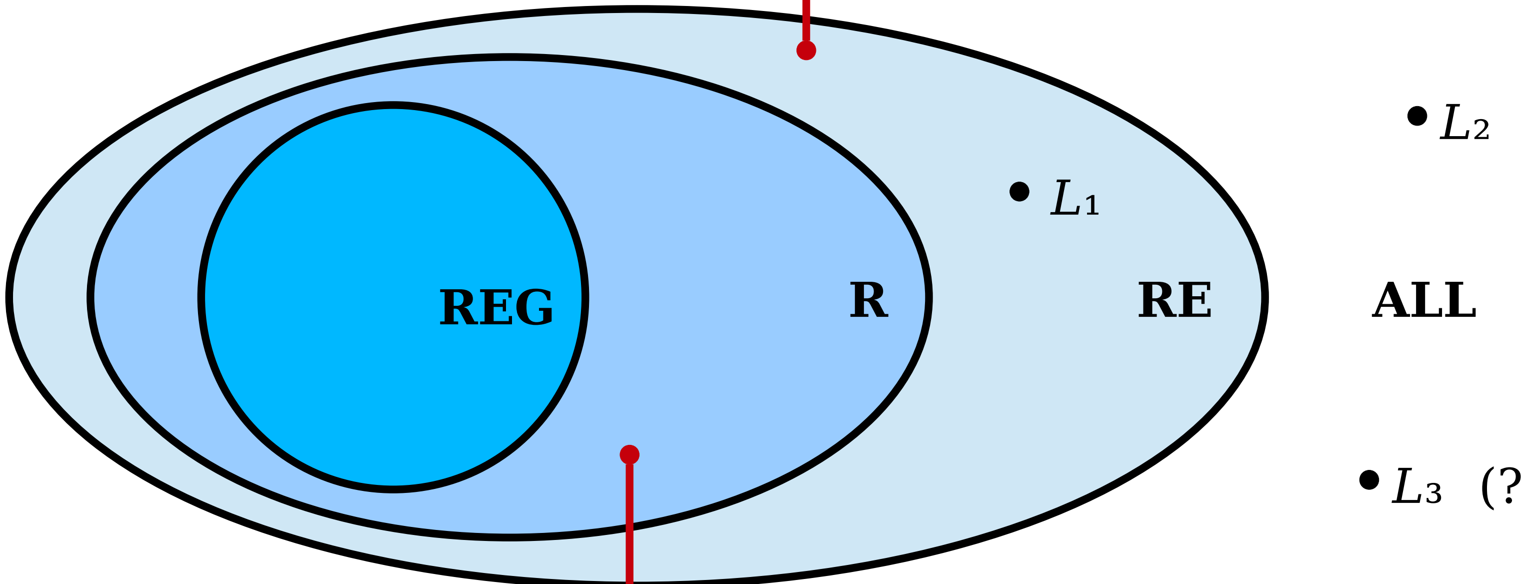

R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

That means that we have a string of the form $a^n b^n$ with at least 2,002 characters in it (at least 1,001 a's and at least 1,001 b's.)

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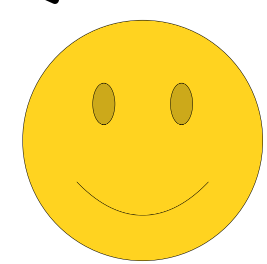

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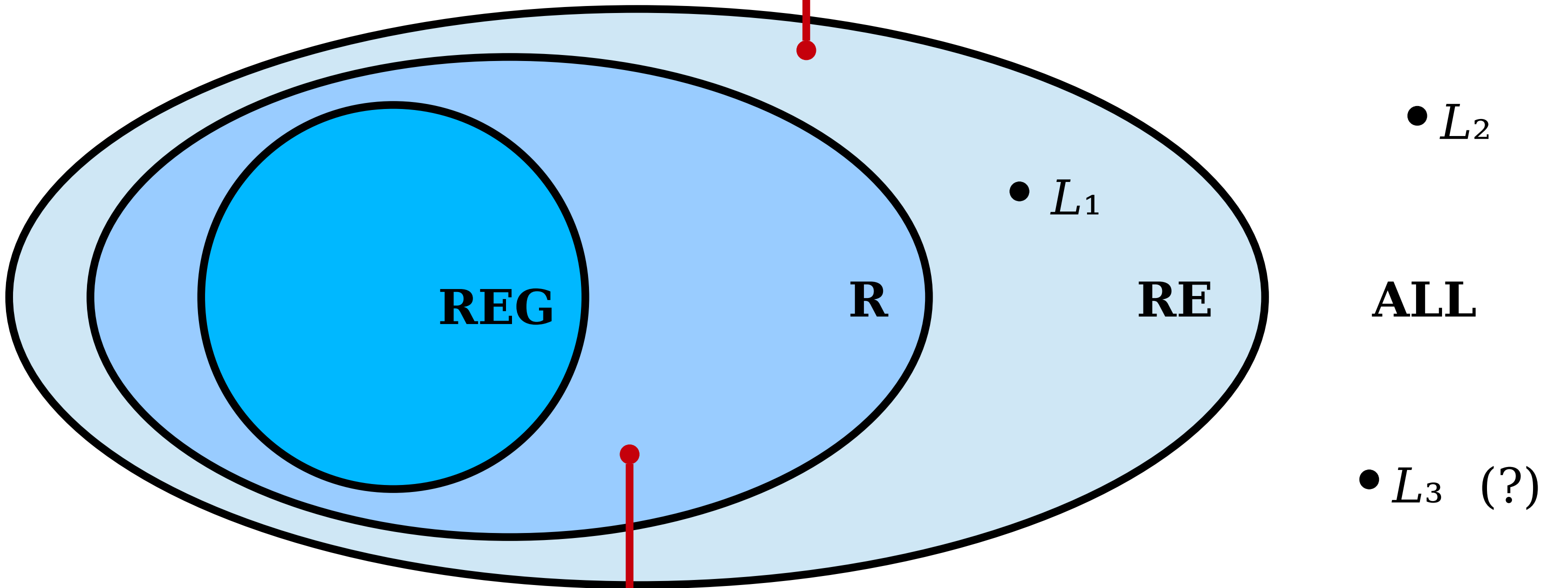

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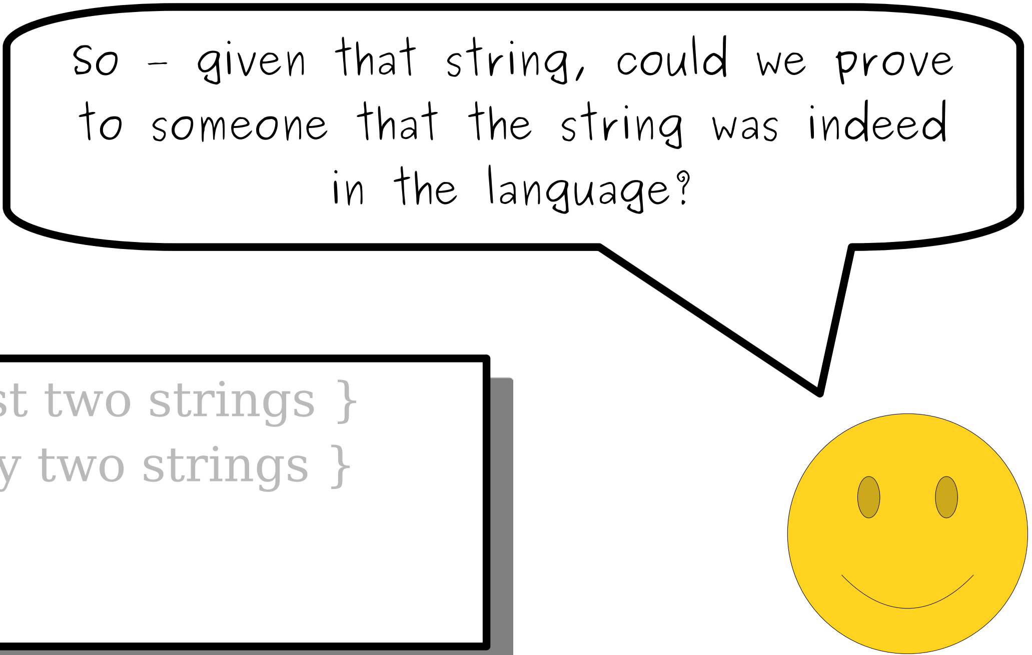

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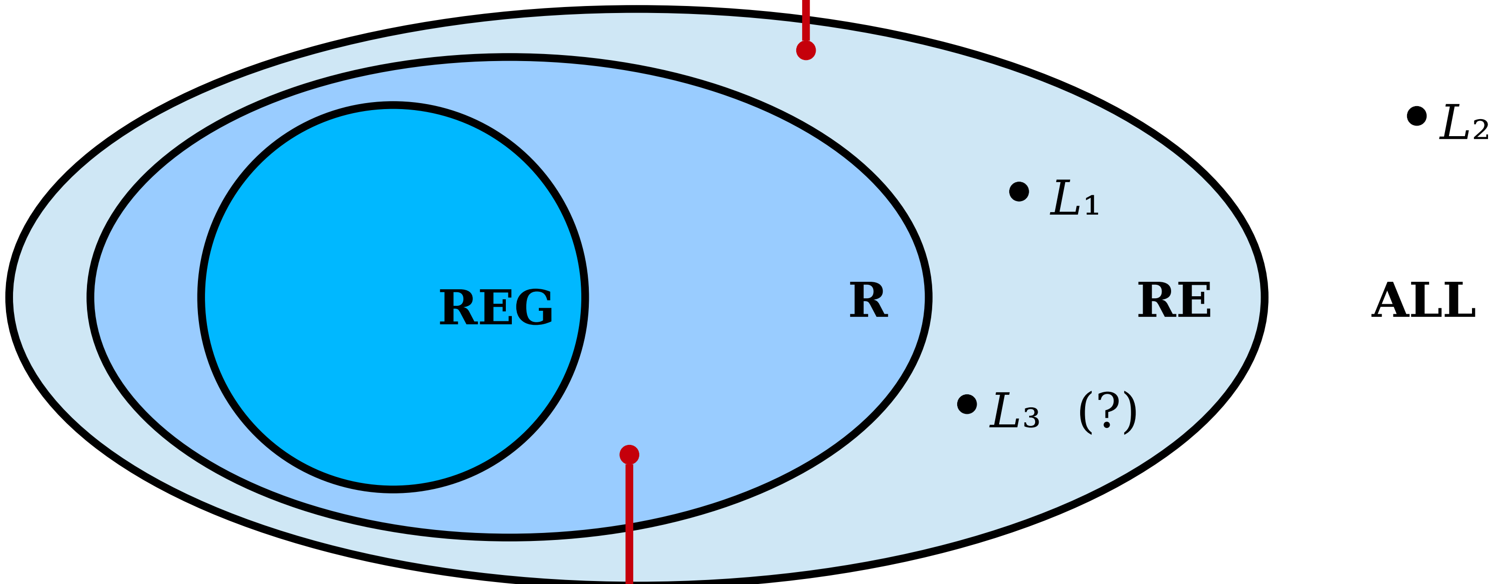

R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

Sure! We could just count up the a's, count up the b's, show that there are the same number, and show that there's at least 1,000.

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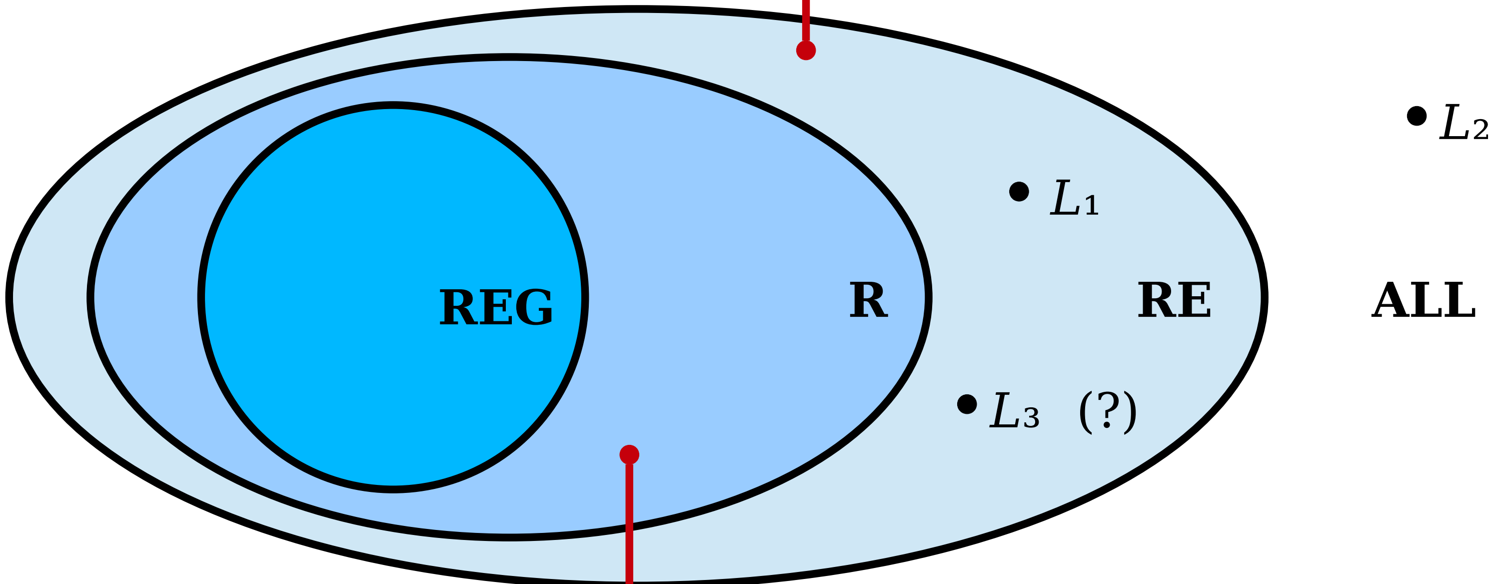

R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

Next, let's ask the follow-up question to see if L_3 is in **R**. If we had a string not in the language, could we prove it?

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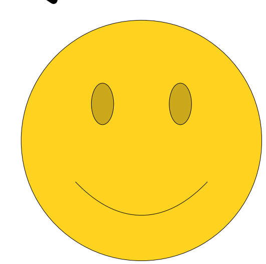

RE: Languages with Verifiers

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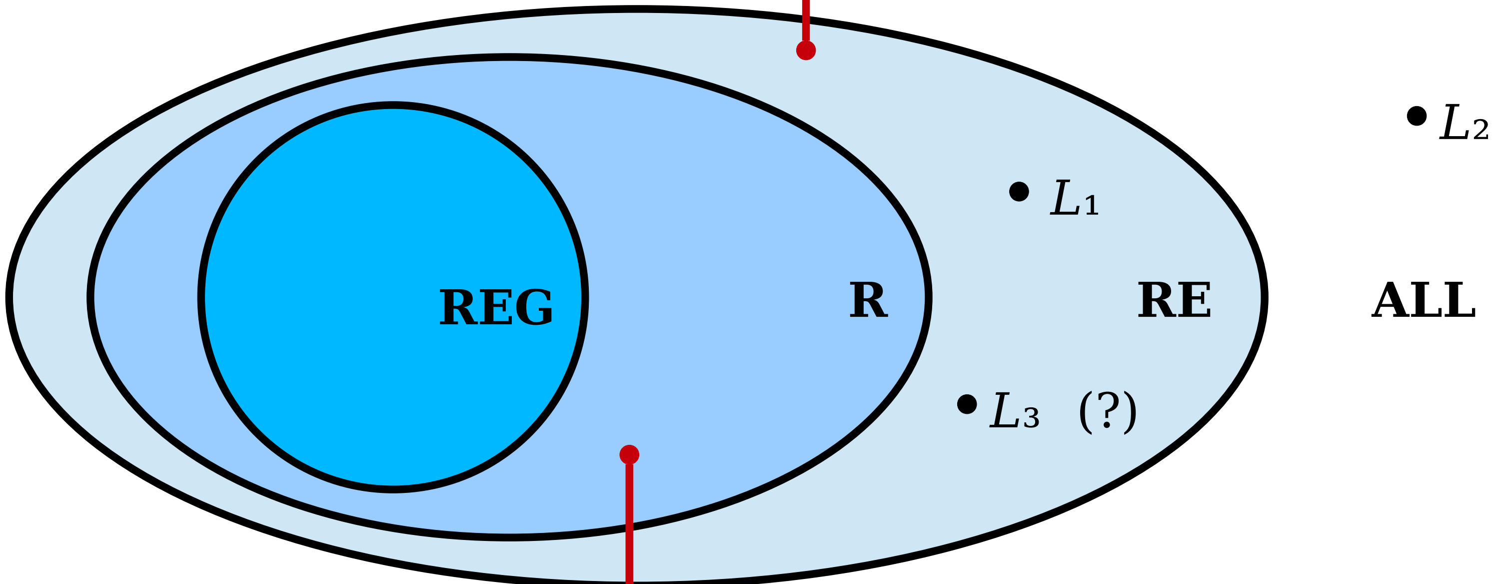

R: Languages with Deciders

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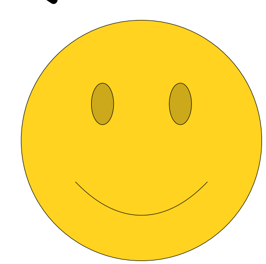
There are a lot of cases to check if the string ends up not being in the language.



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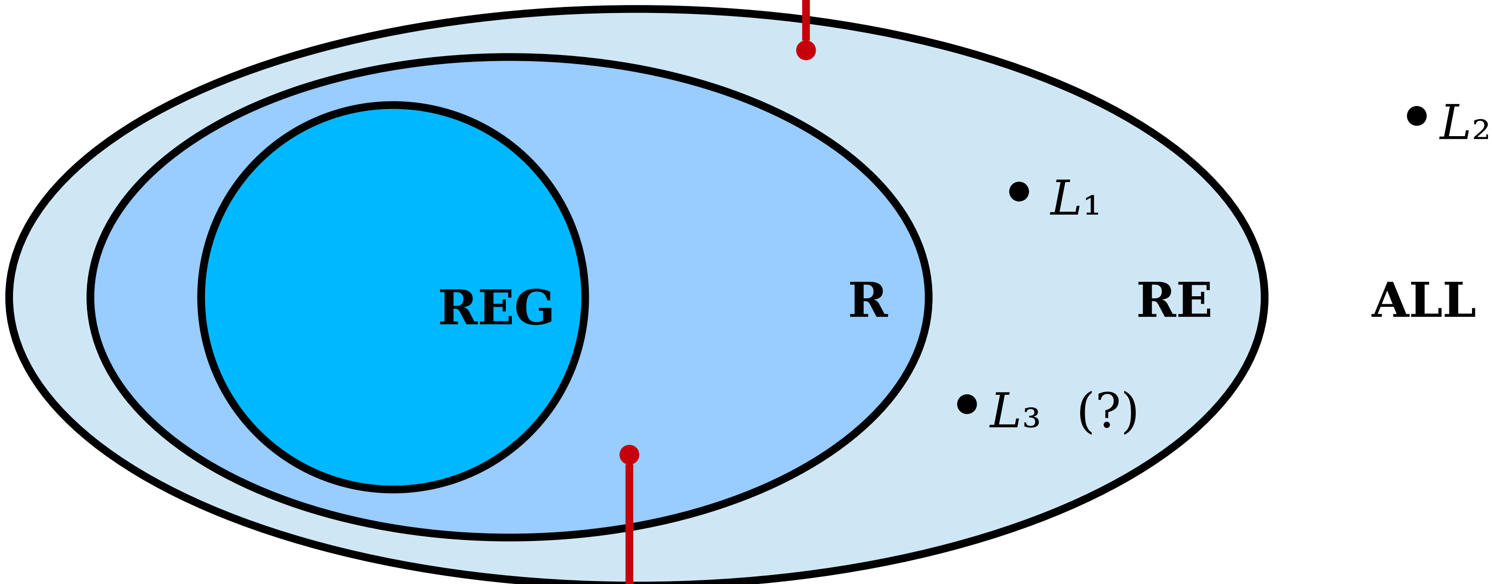

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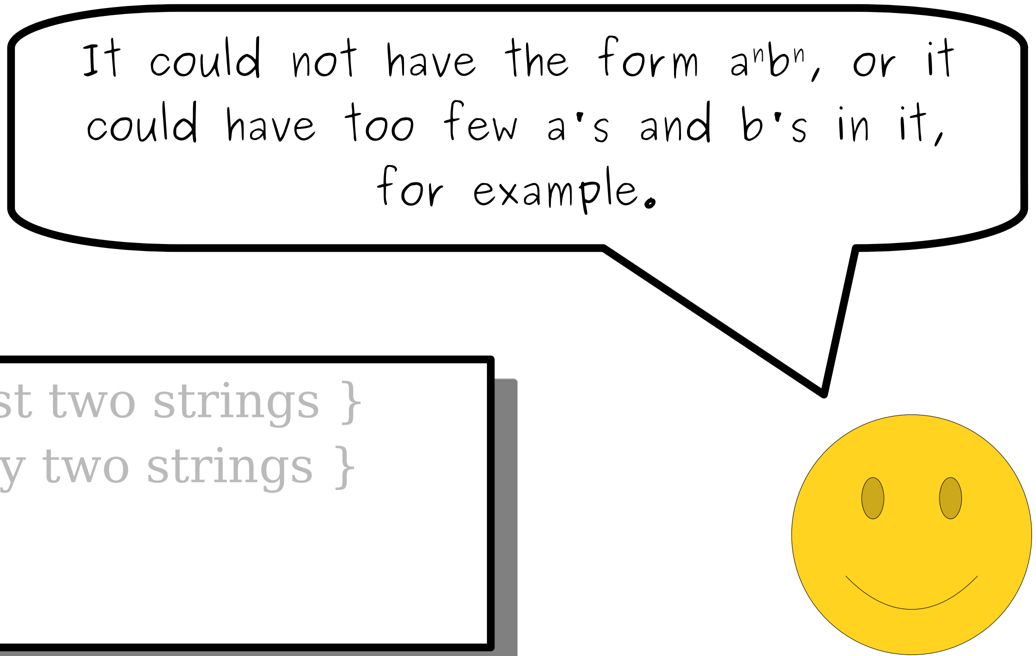

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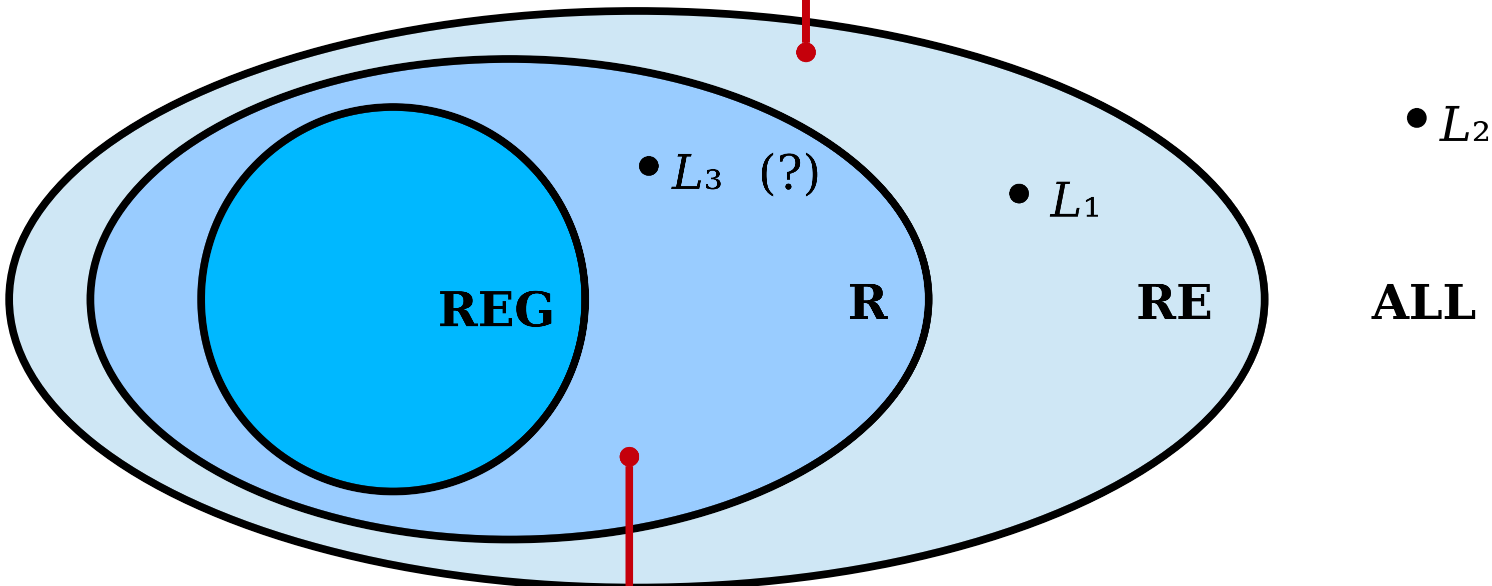

R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

However, all of those cases are really easy to check. We either show that it has the wrong form or show that it doesn't have enough characters in it.

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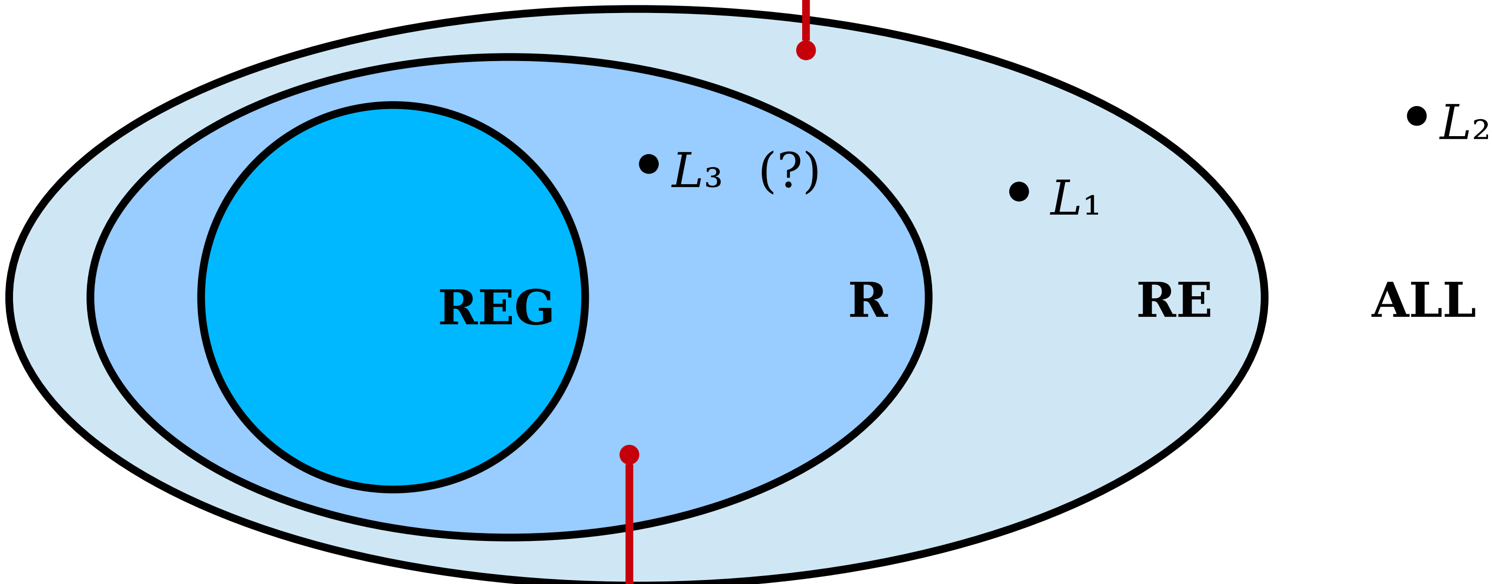
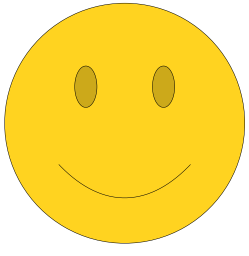


R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

Okay, things are looking good here! We know that this language is decidable. As our final step, we need to ask whether or not it's regular.



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RE: Languages with Verifiers

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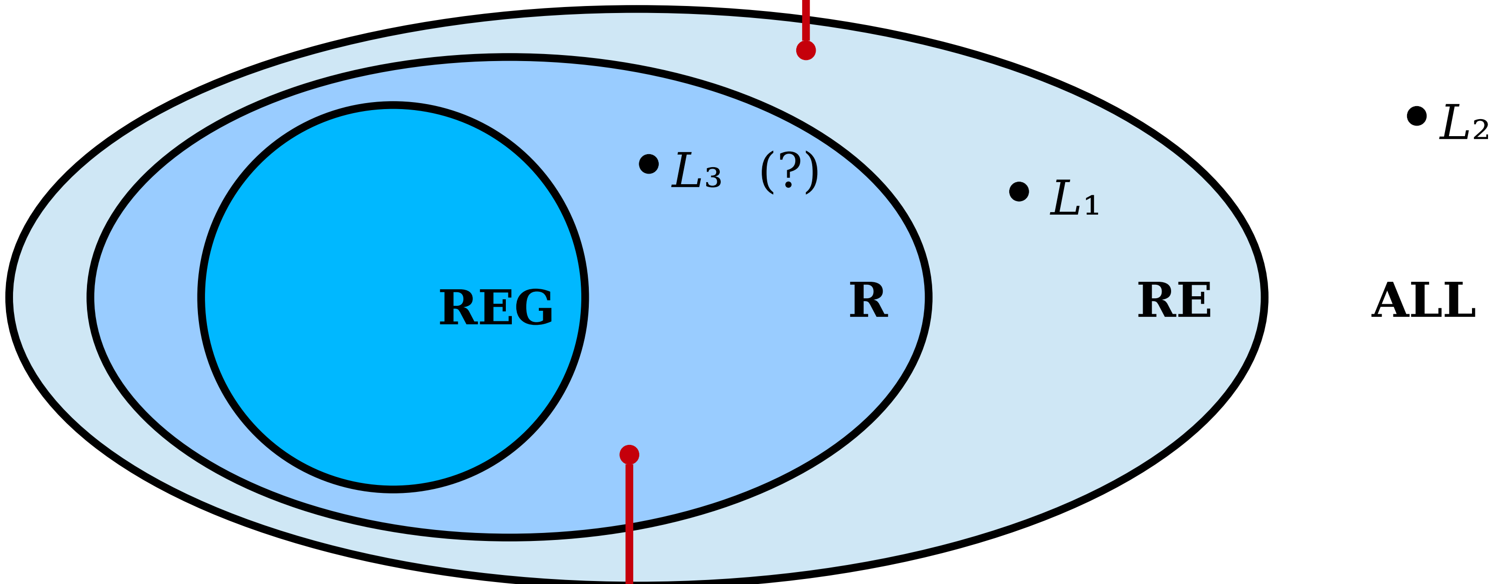

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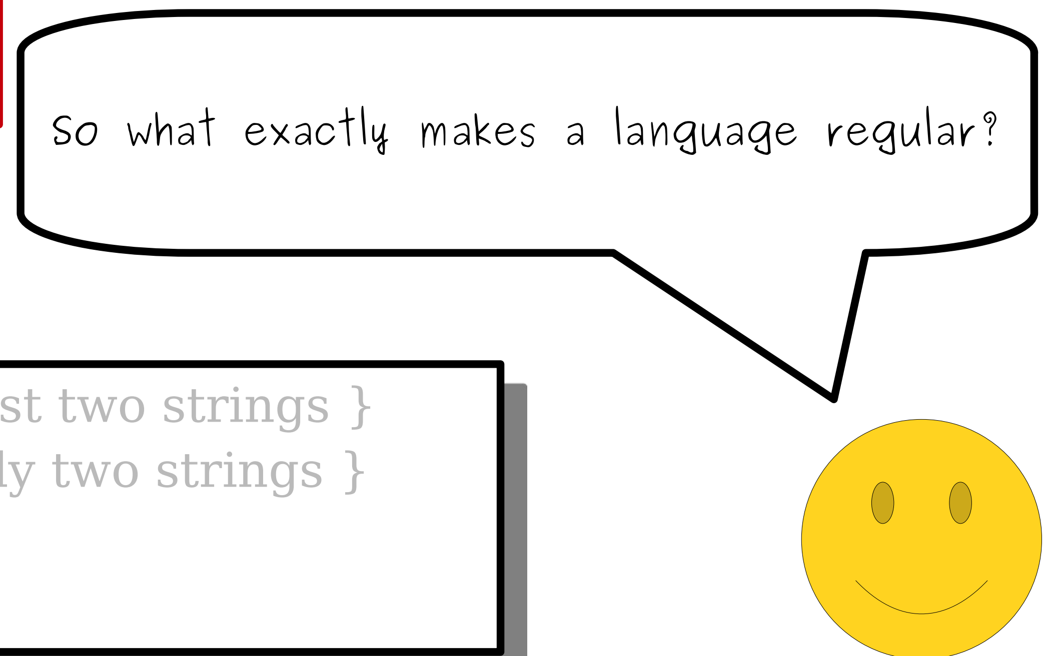

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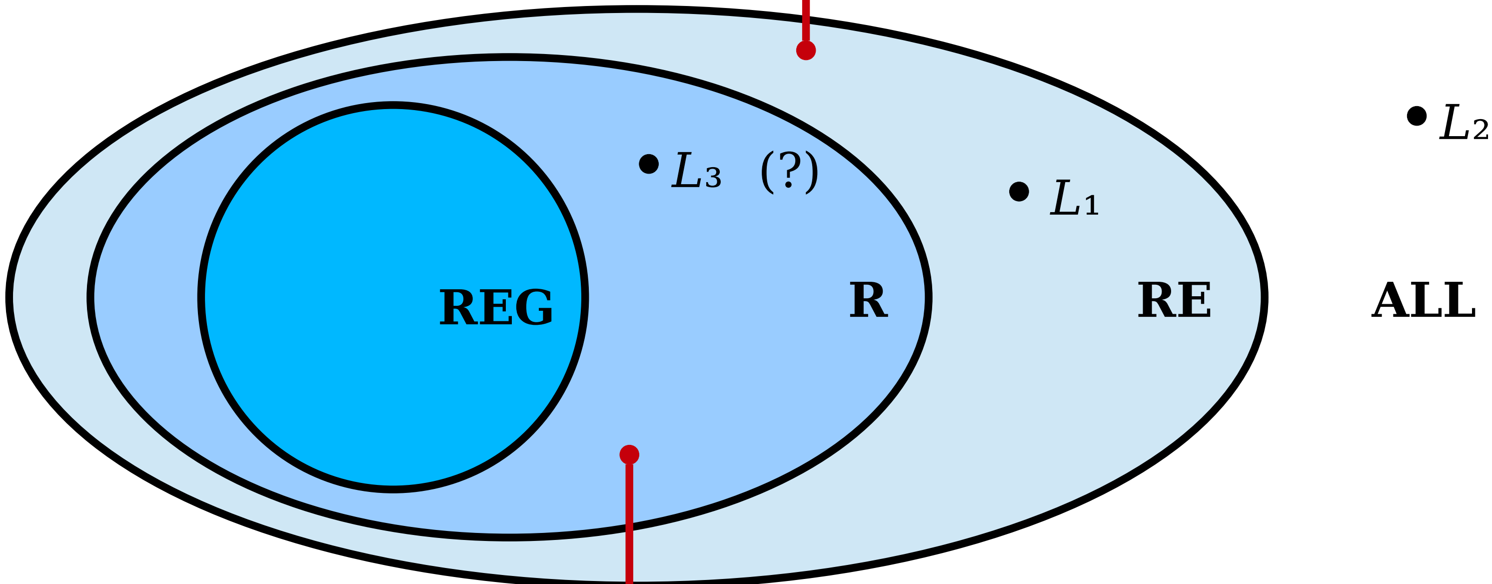
$L_3 = \{ \texttt{a}^n\texttt{b}^n \mid n \in \mathbb{N}$ and $n > 1000 \}$

$L_4 = \{ \texttt{a}^n\texttt{b}^n \mid n \in \mathbb{N}$ and $n \leq 1000 \}$

RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

We have a ton of different definitions for regular languages – they're the languages of DFAs, NFAs, and regexes.

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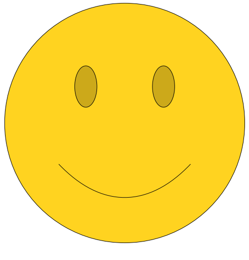
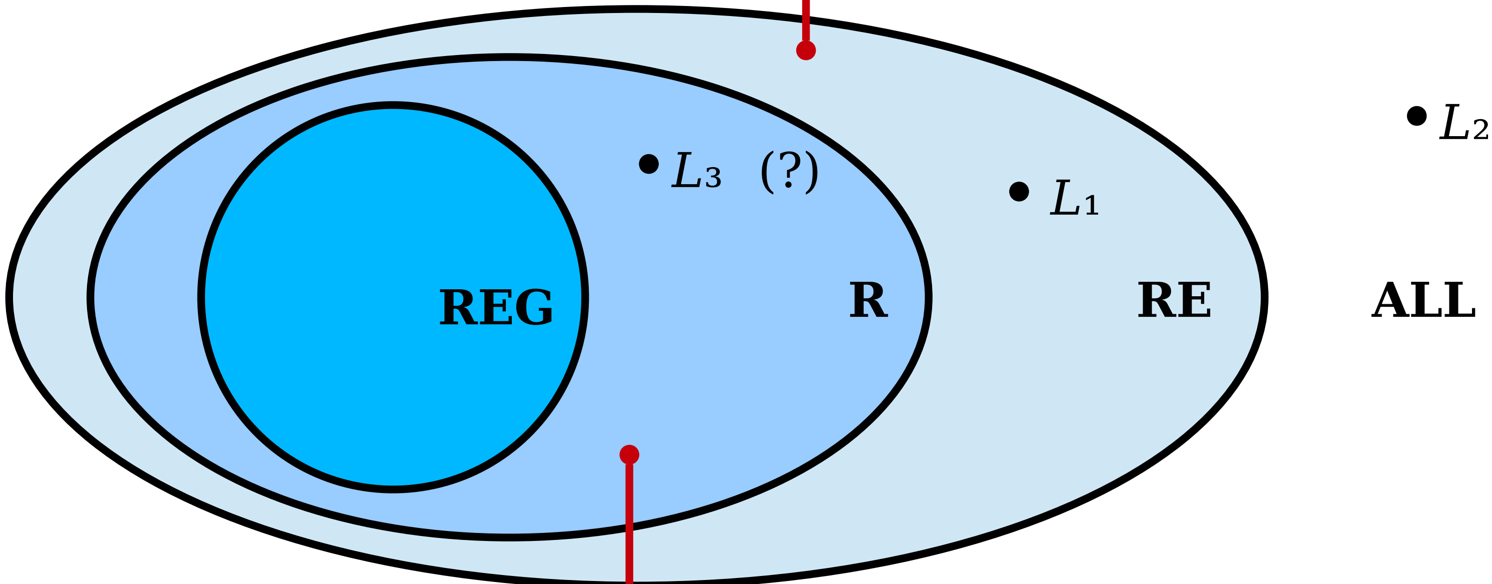


RE: Languages with Verifiers

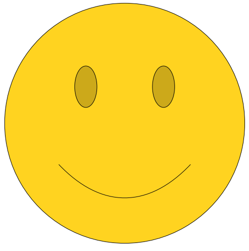
Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

But, as with **R** and **RE**, I think there's a much better intuition to have about the regular languages that makes it easier to see whether something is regular.



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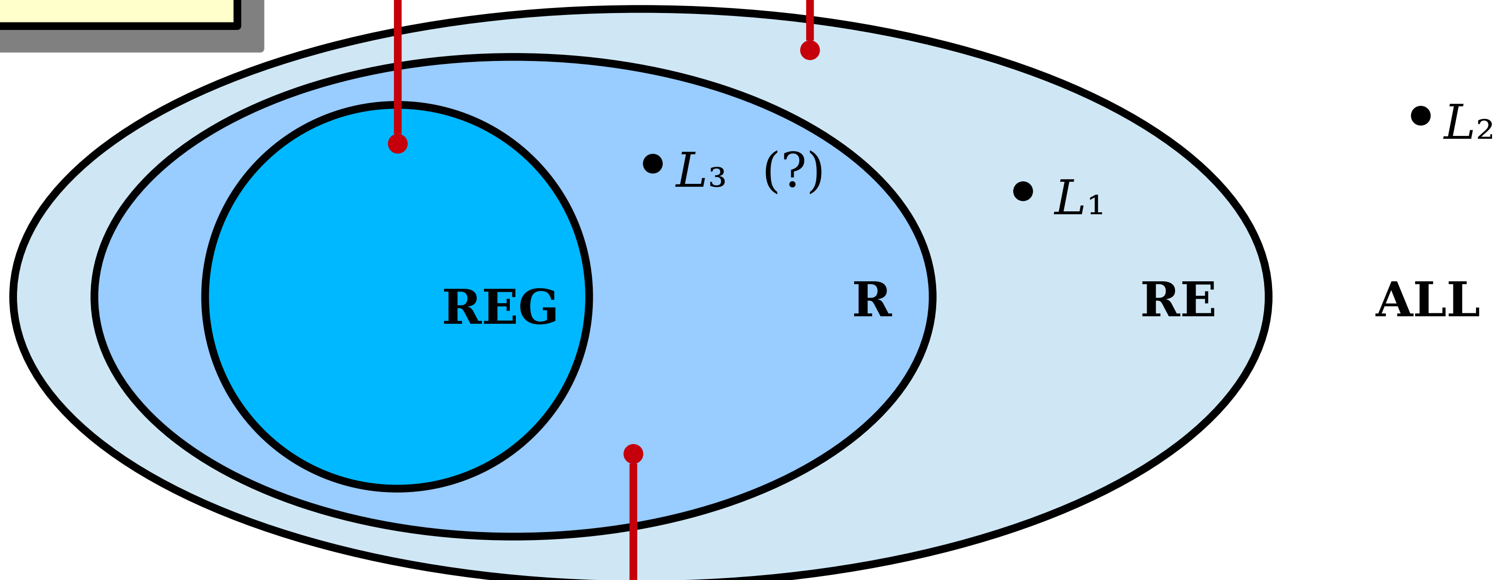
REG: Problems Solvable with Finite Memory

RE: Languages with Verifiers

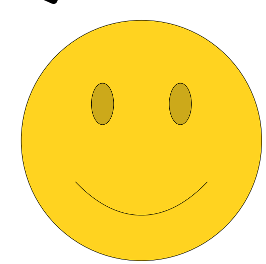
Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

Specifically, the regular languages really correspond to problems that you can solve in finite memory. (This is the same intuition we used to find nonregular languages for the first time.)



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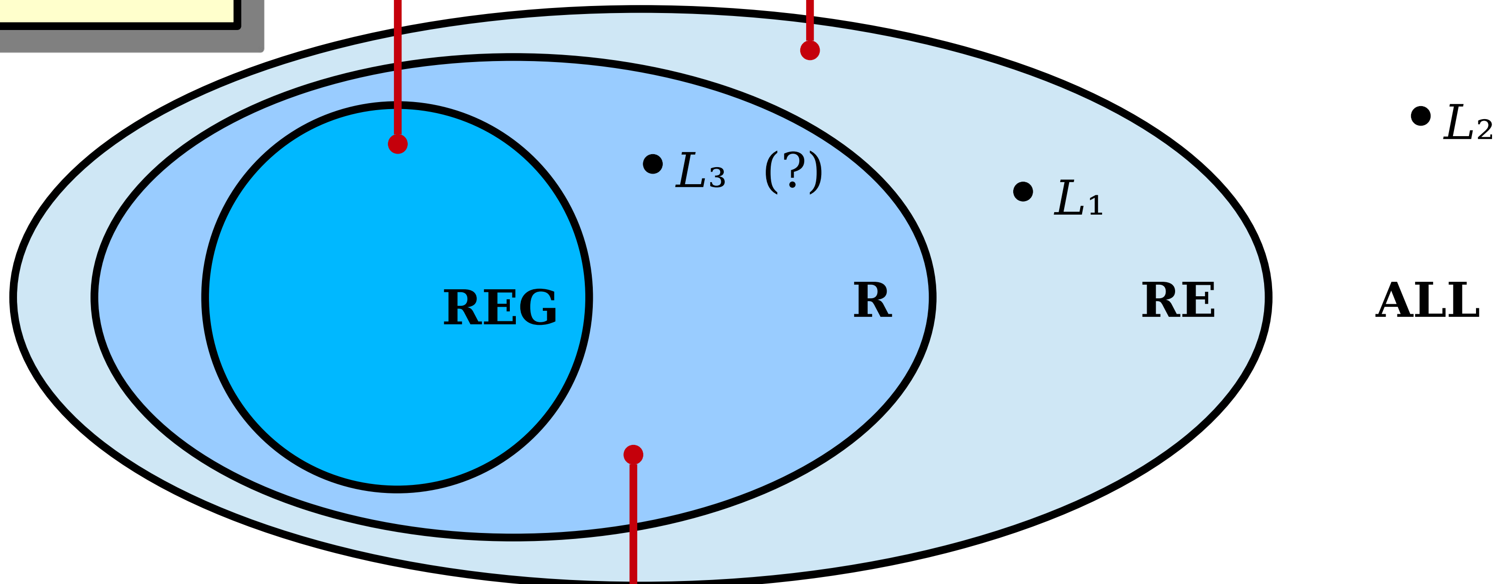
REG: Problems Solvable with Finite Memory

RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

If you're trying to determine whether a decidable language happens to be regular, think about how much information you need to remember about the input string.

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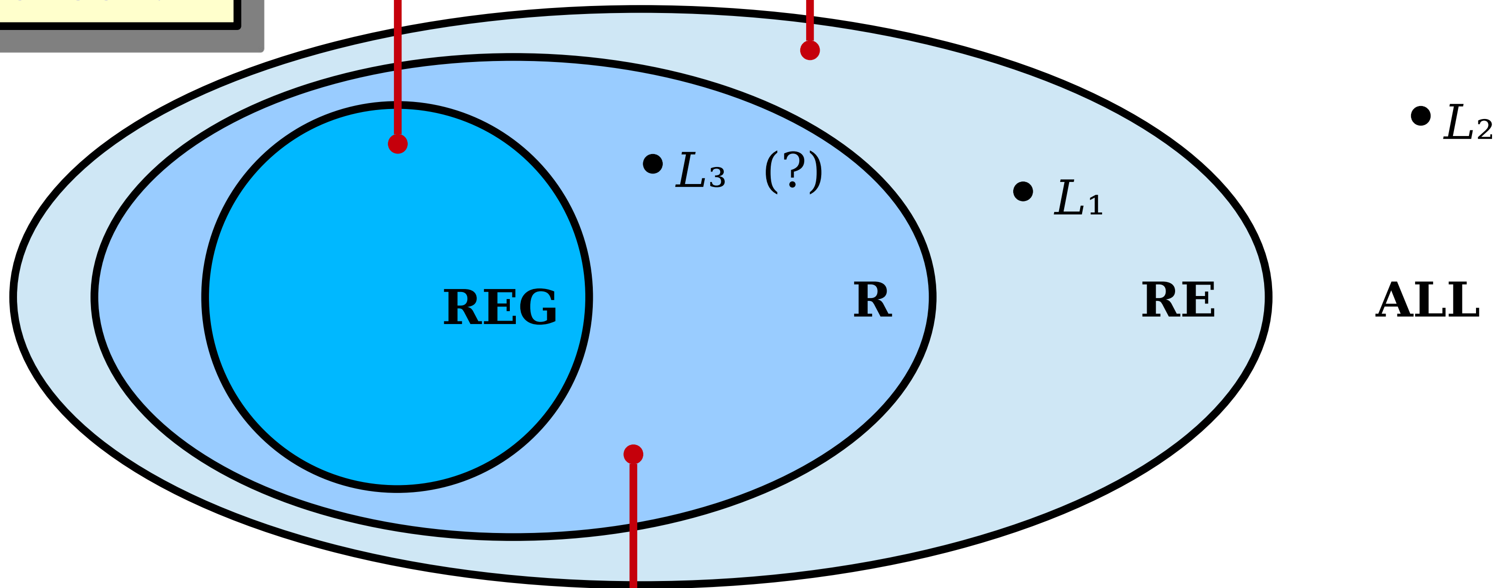
REG: Problems Solvable with Finite Memory

Are there are finitely many cases to check?

RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

If you only need to remember one of finitely many pieces of information, then the language is almost certainly regular, even if you can't envision a clean DFA or regex for it.

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REG: Problems Solvable with Finite Memory

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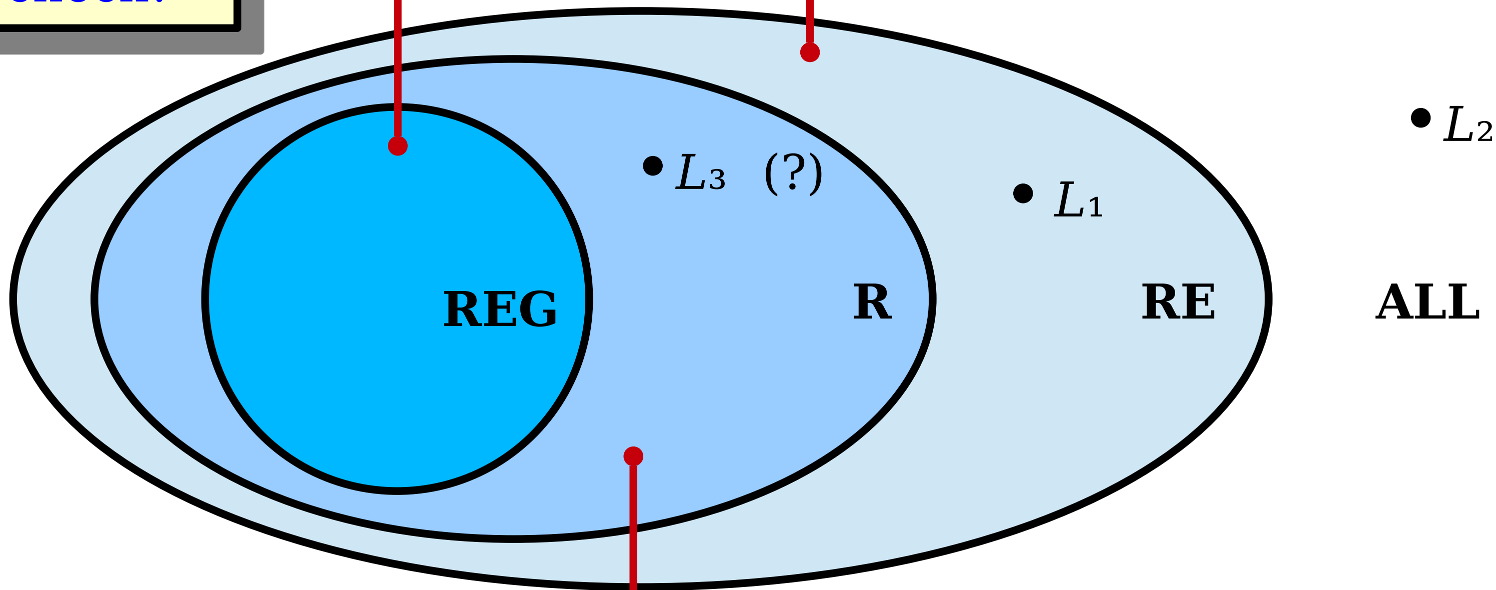
RE: Languages with Verifiers

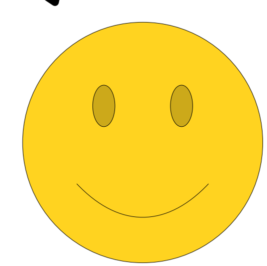
Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

So let's think about this here. What information do we need to keep track of?



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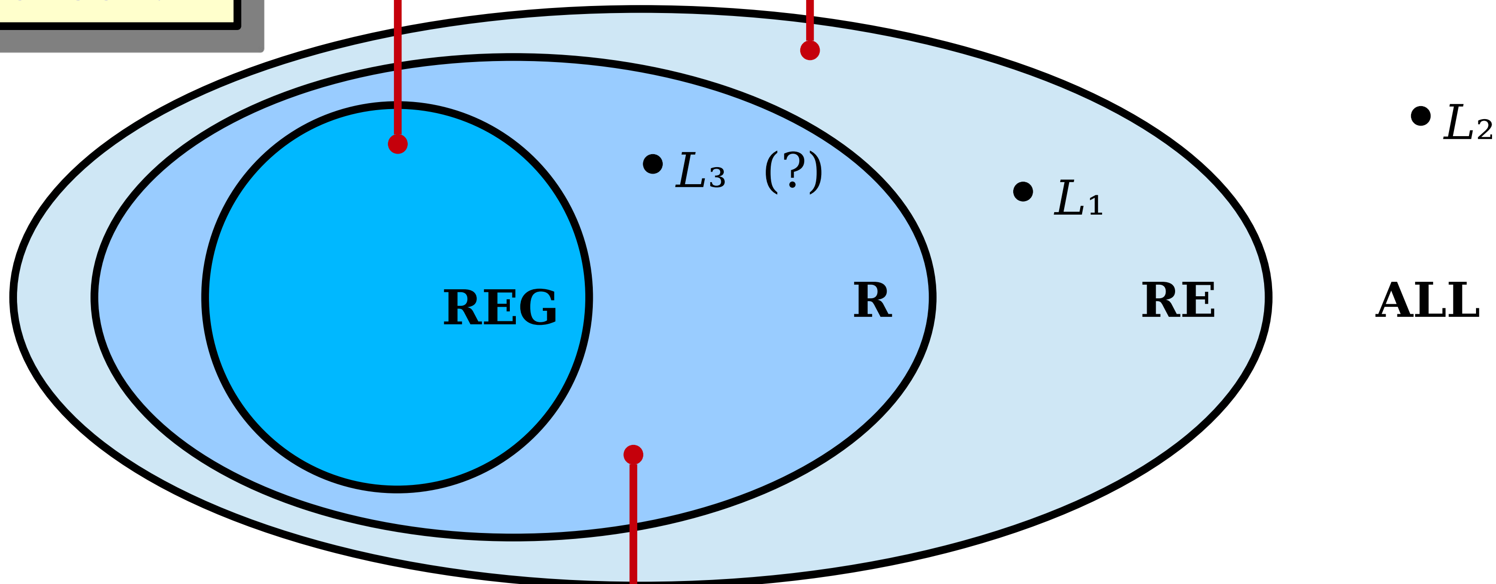
REG: Problems Solvable with Finite Memory

Are there are finitely many cases to check?

RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

Fundamentally, we'd have to keep track of how many a's we've seen, since if we can't do that, we can't match it against the number of b's.

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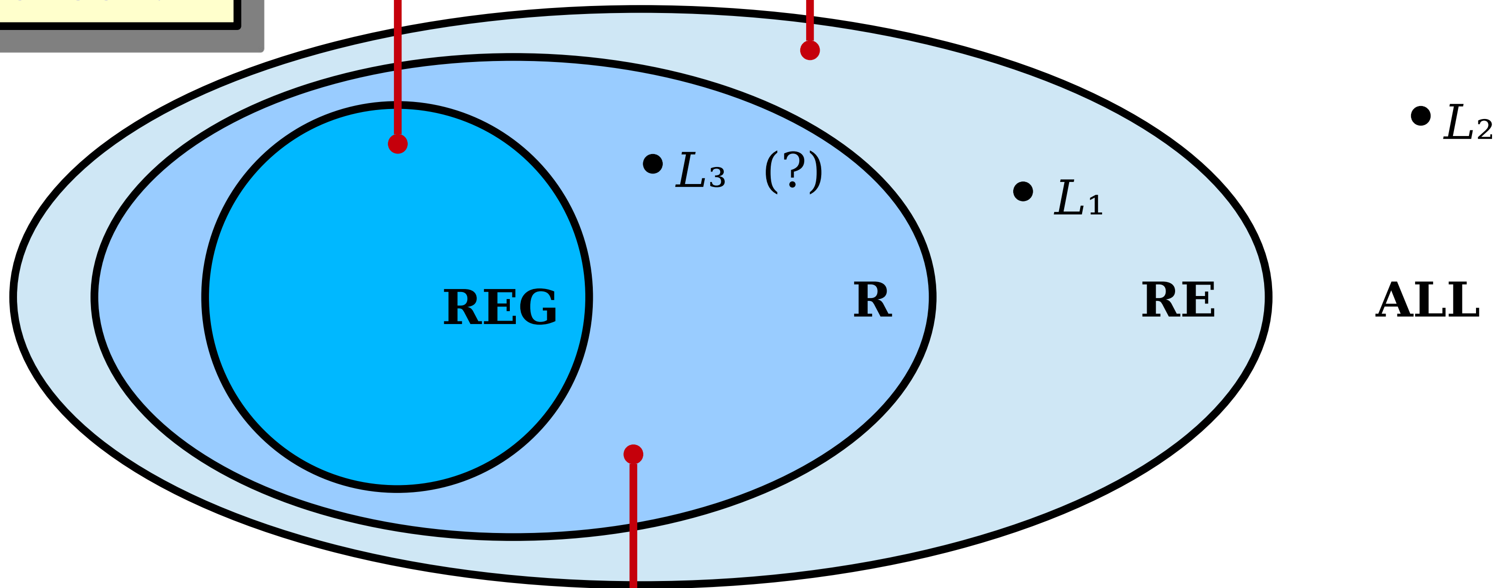
REG: Problems Solvable with Finite Memory

Are there are finitely many cases to check?

RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

That's a problem: there are infinitely many possible choices for the number of a's that we'd have to remember, and we can't remember which number we've seen with finitely many states!

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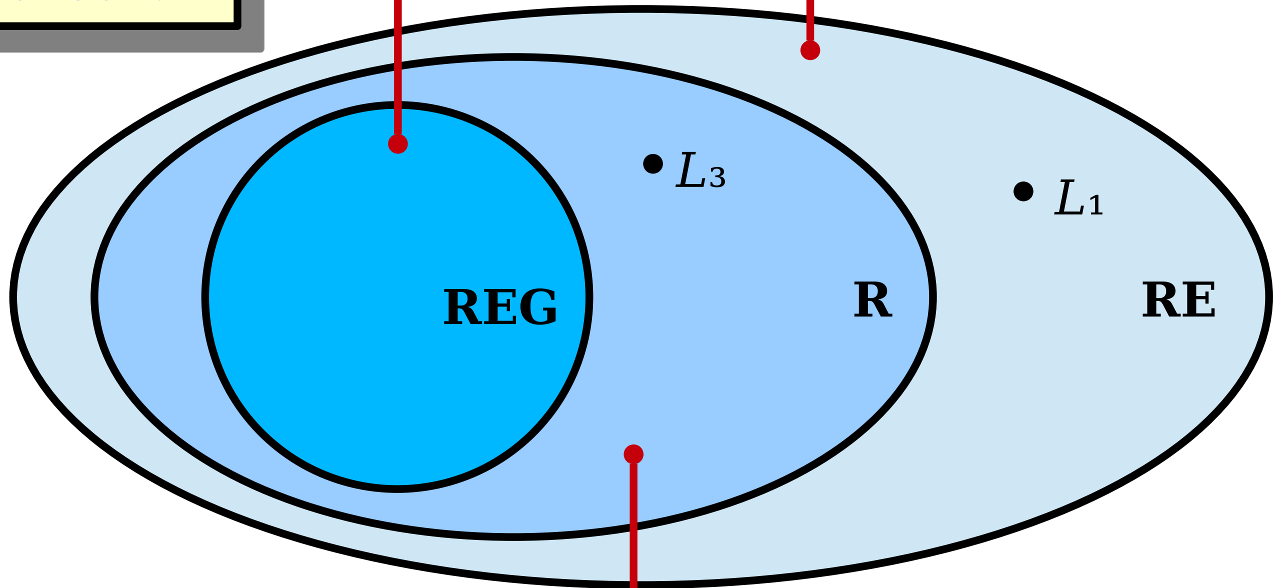
REG: Problems Solvable with Finite Memory

Are there are finitely many cases to check?

RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

So this gives us the intuition that L_3 is almost certainly going to be nonregular.

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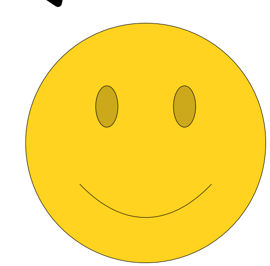
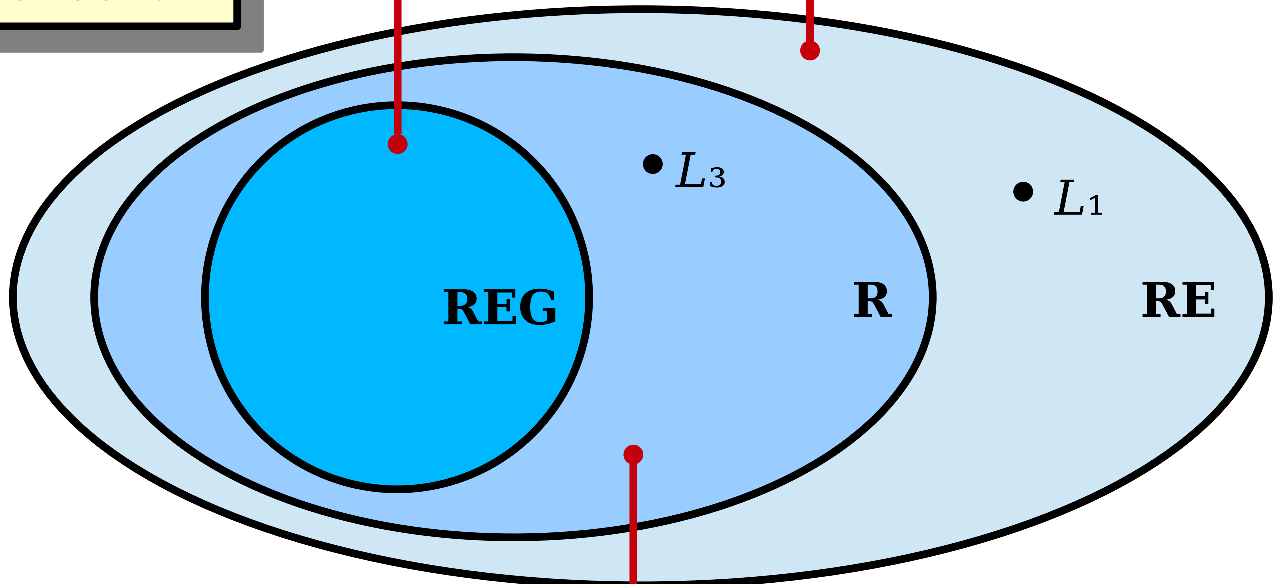


REG: Problems Solvable with Finite Memory

Are there are finitely many cases to check?

RE: Languages with Verifiers

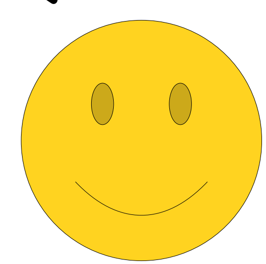
Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

You can formally prove this by using the Myhill–Nerode theorem. I highly recommend it – it's good practice!



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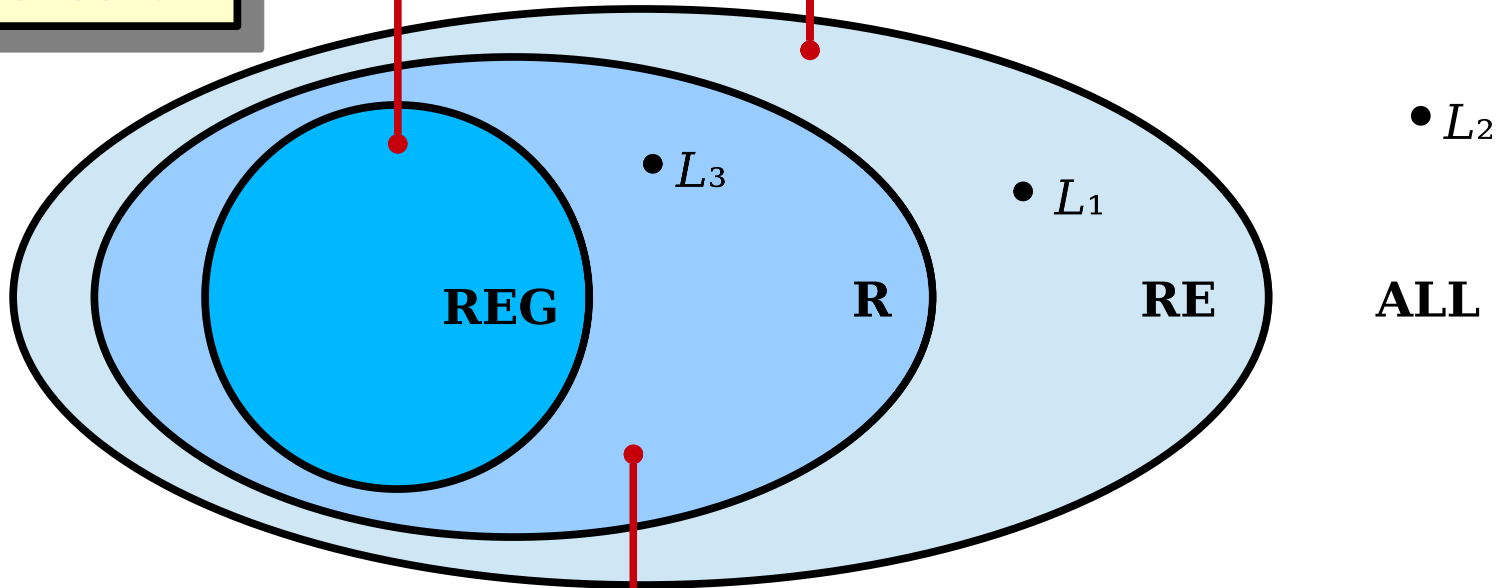
REG: Problems Solvable with Finite Memory

Are there are finitely many cases to check?

RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

so – what did we learn here?

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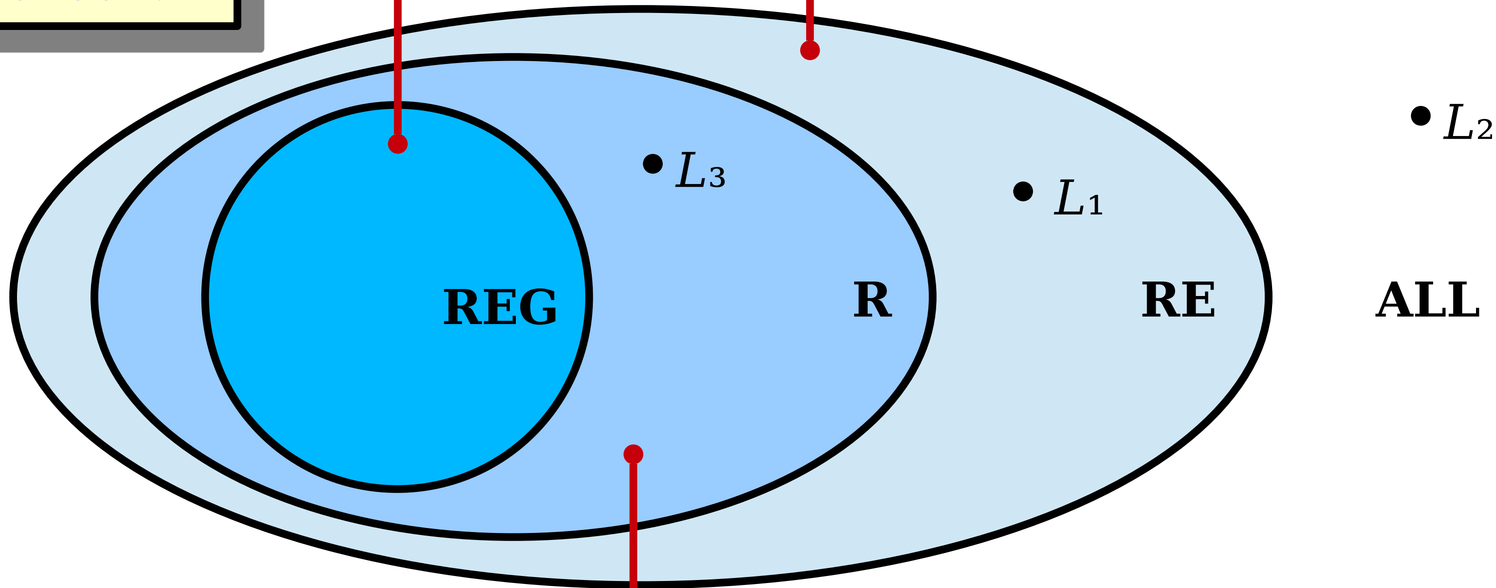
REG: Problems Solvable with Finite Memory

Are there are finitely many cases to check?

RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

We've seen how to use our key intuition for regular languages – they're languages you can solve in finite space – to check whether something is regular.

L_1 = { ⟨*M*⟩ | *M* is a TM that accepts at least two strings }

L_2 = { ⟨*M*⟩ | *M* is a TM that accepts exactly two strings }

$L_3 = \{ \mathtt{a}^n\mathtt{b}^n \mid n \in \mathbb{N} \text{ and } n > 1000 \}$

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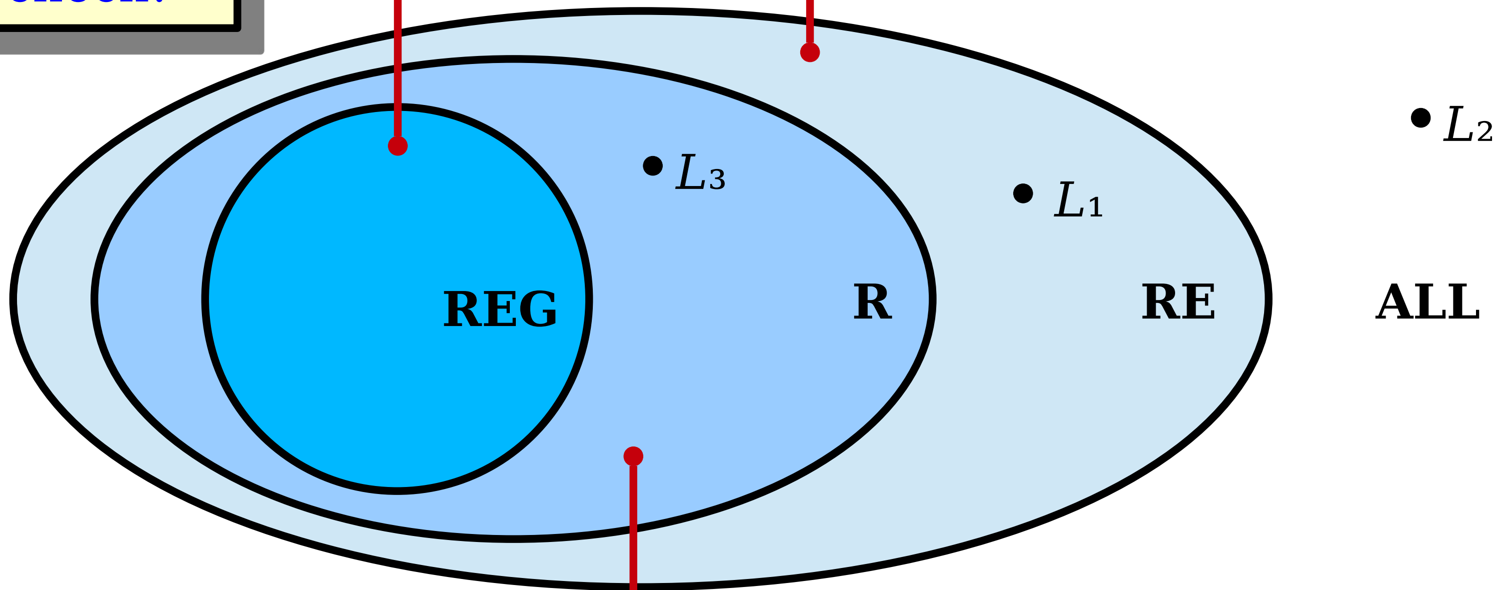
REG: Problems Solvable with Finite Memory

Are there are finitely many cases to check?

RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

With all that said and done, let's move on to our last language here.

L_1 = { ⟨M⟩ | M is a TM that accepts at least two strings }

L_2 = { ⟨M⟩ | M is a TM that accepts exactly two strings }

L_3 = { $\mathtt{a}^n\mathtt{b}^n$ | $n \in \mathbb{N}$ and $n > 1000$ }

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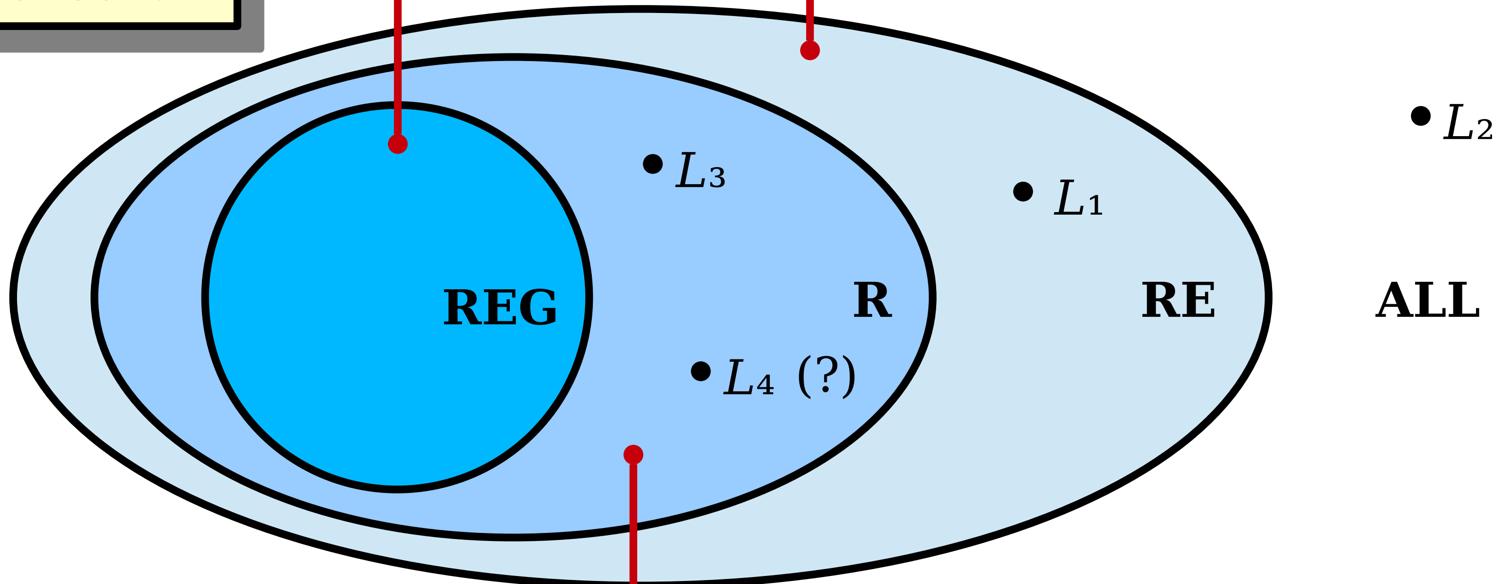
REG: Problems Solvable with Finite Memory

Are there are finitely many cases to check?

RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

While normally we've talked about starting from the outside and moving inward, for this language I think you can probably see that this is going to be decidable, so let's start it there.

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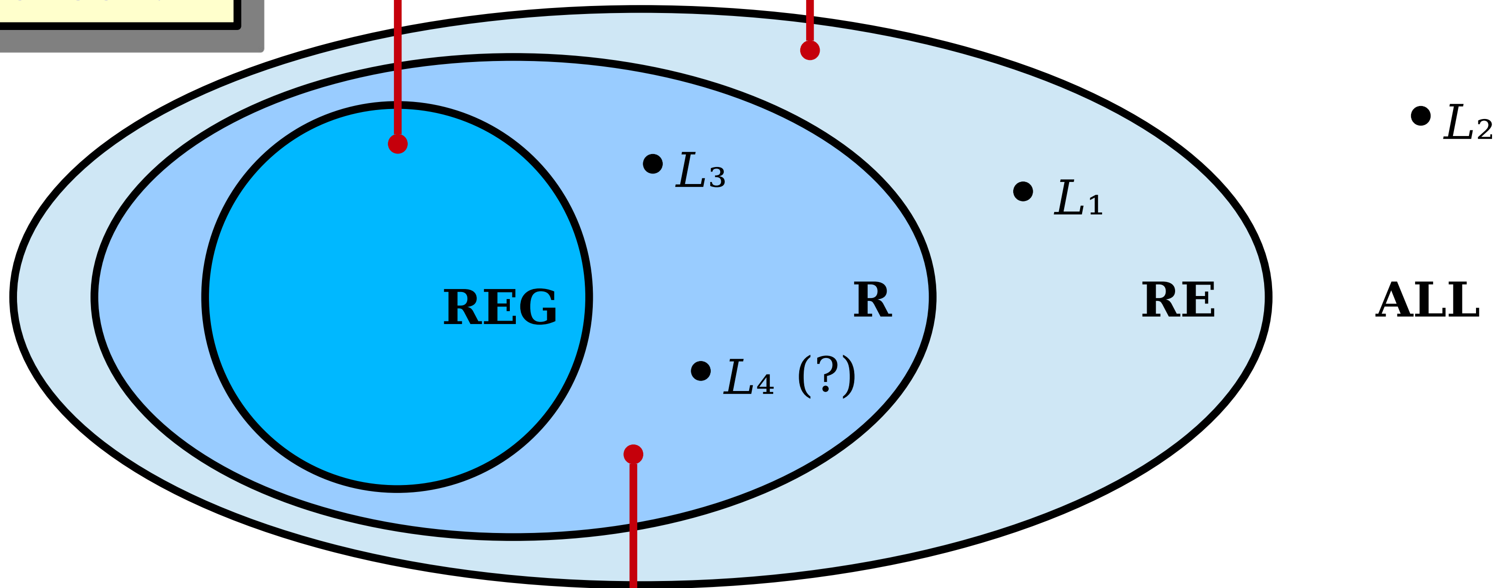
REG: Problems Solvable with Finite Memory

Are there are finitely many cases to check?

RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

The question now is whether it's regular or not.

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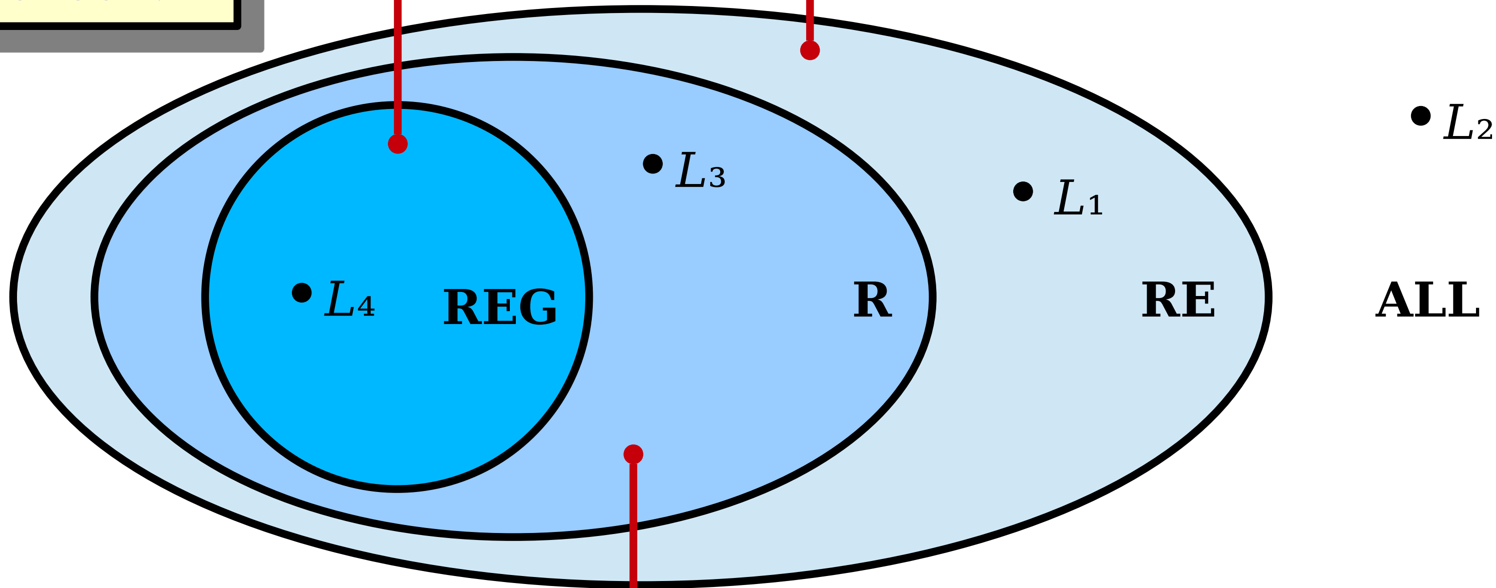
REG: Problems Solvable with Finite Memory

Are there are finitely many cases to check?

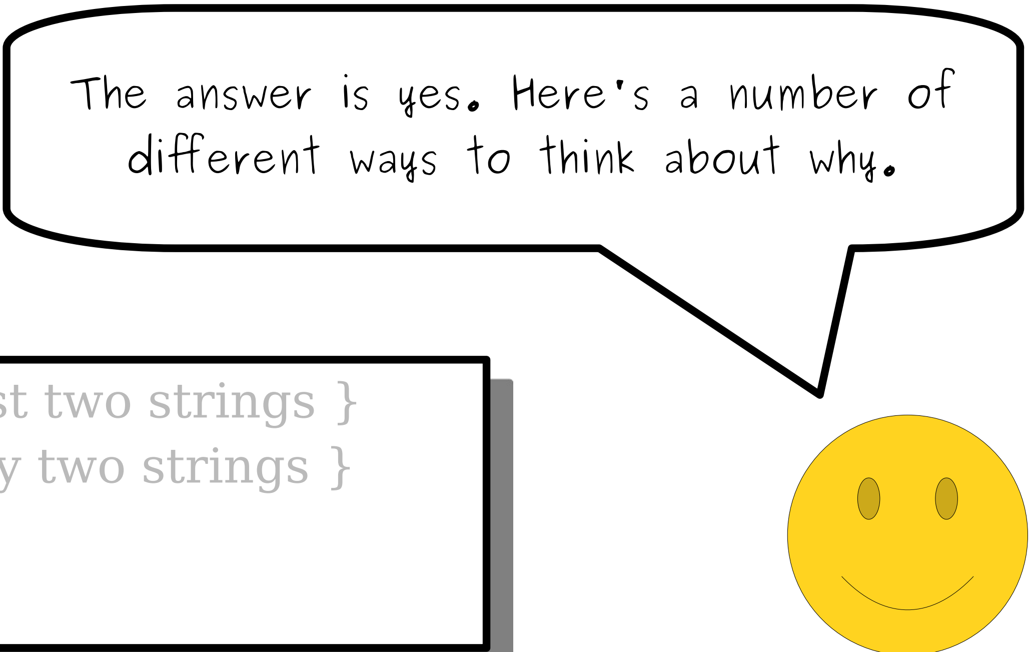
RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?



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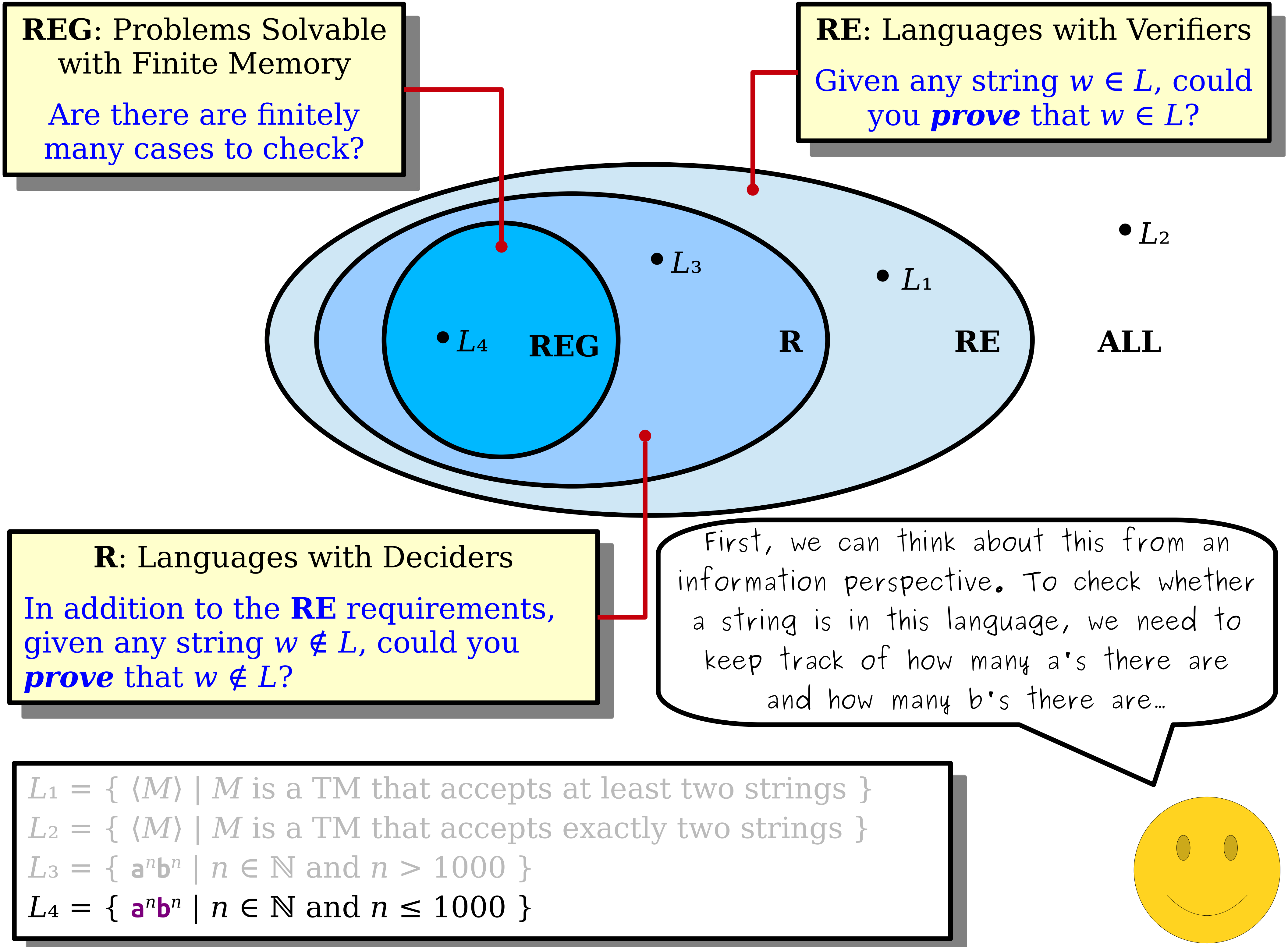

REG: Problems Solvable with Finite Memory
Are there are finitely many cases to check?
RE: Languages with Verifiers
Given any string $w \in L$, could you ***prove*** that $w \in L$?
L_2
L_3
L_1
L_4
REG
R
RE
ALL
R: Languages with Deciders
In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?
First, we can think about this from an information perspective. To check whether a string is in this language, we need to keep track of how many a's there are and how many b's there are…
L_1 = { ⟨M⟩ | M is a TM that accepts at least two strings }
L_2 = { ⟨M⟩ | M is a TM that accepts exactly two strings }
L_3 = { $\mathtt{a}^n\mathtt{b}^n$ | $n \in \mathbb{N}$ and $n > 1000$ }
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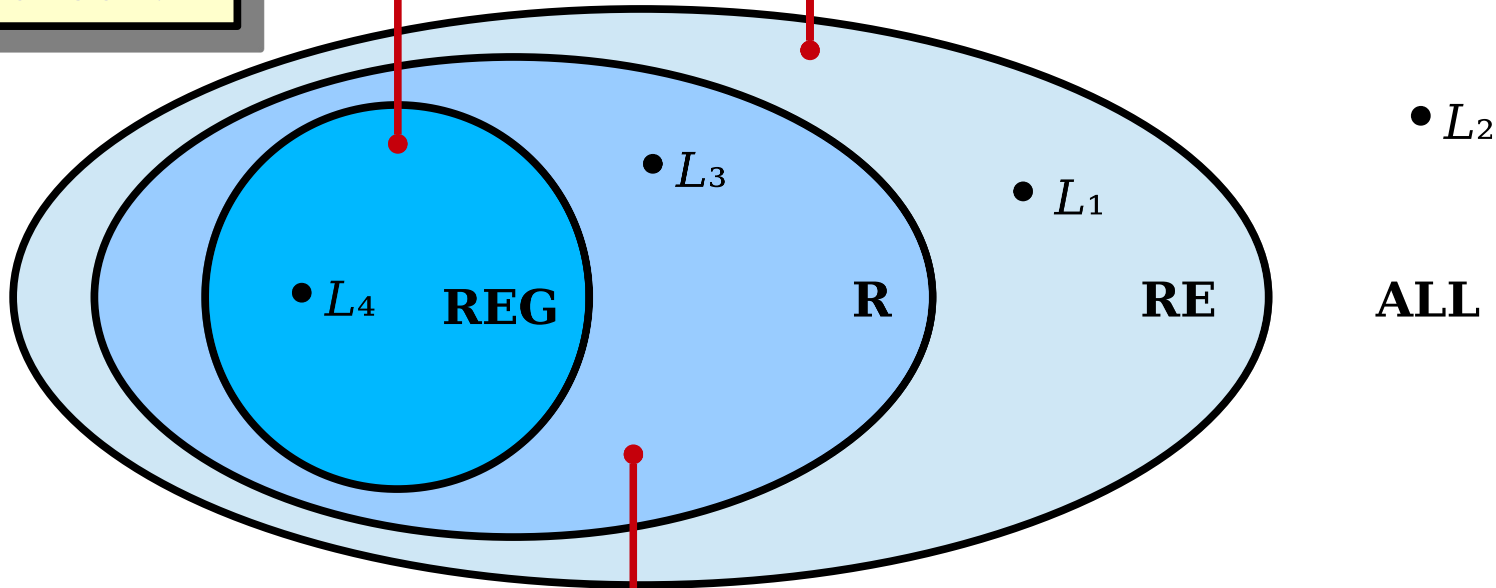
REG: Problems Solvable with Finite Memory

Are there are finitely many cases to check?

RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

…but only up to a point. After we see 1,001 copies of either character, we know that the string isn't in the language.

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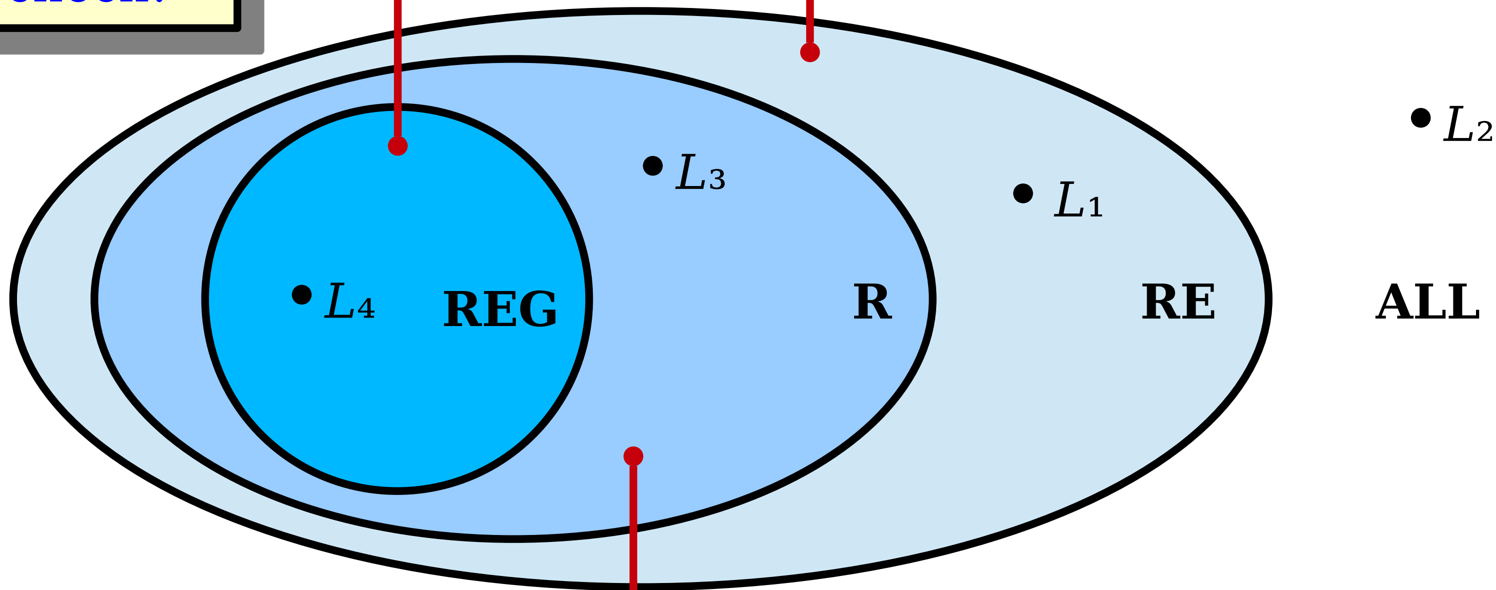
REG: Problems Solvable with Finite Memory

Are there are finitely many cases to check?

RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

This means that we just need to remember how many a's and b's we've seen (within the limits) and whether we're still reading a's or b's.

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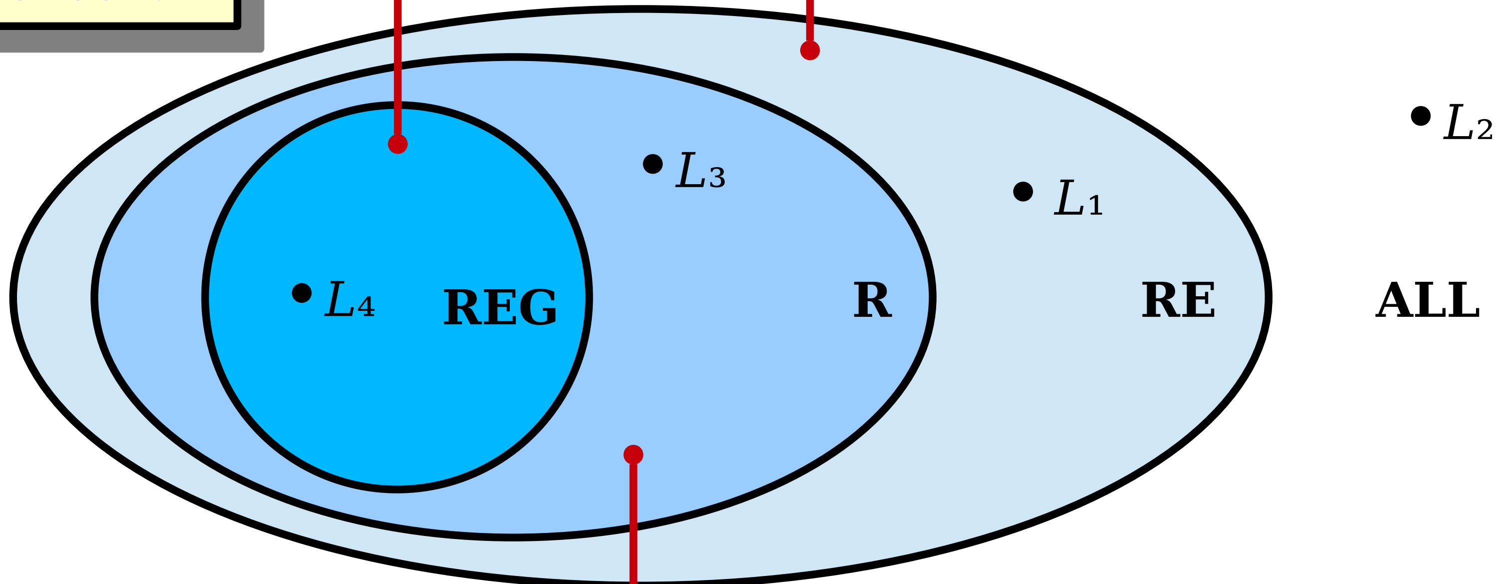
REG: Problems Solvable with Finite Memory

Are there are finitely many cases to check?

RE: Languages with Verifiers

Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

That means we only need a finite amount of information to decide whether a string is in the language, so using our intuition for the regular languages, this one willl be regular.

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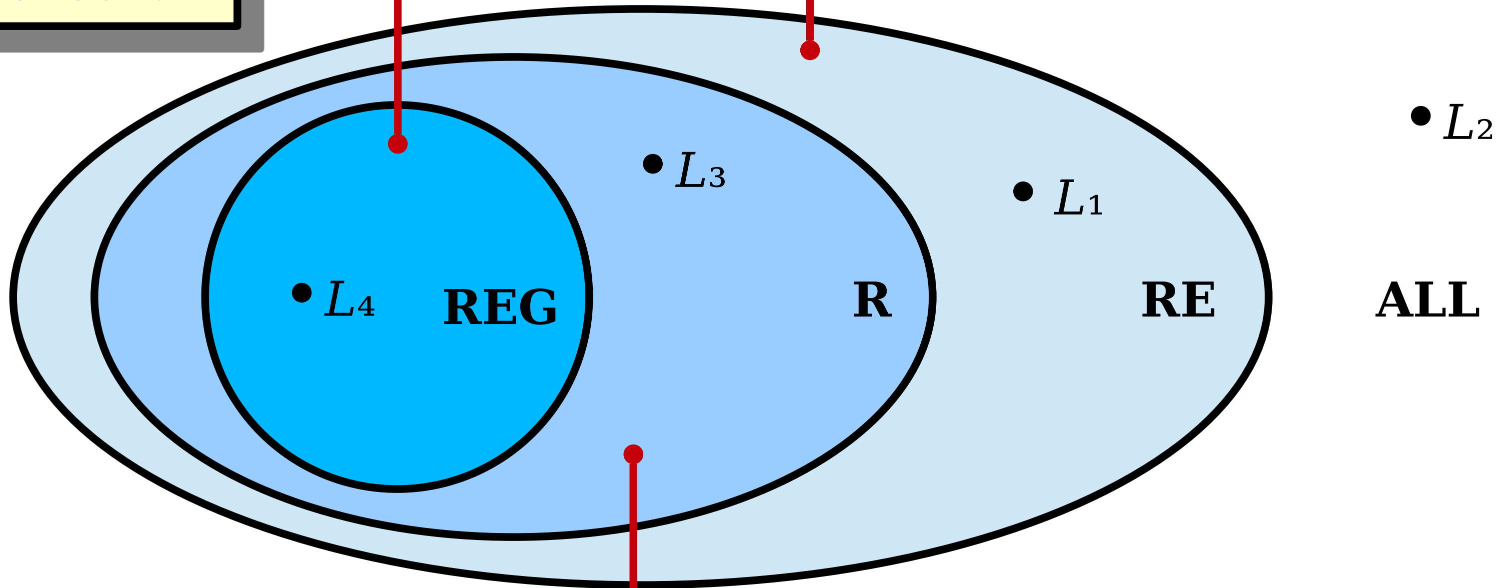
REG: Problems Solvable with Finite Memory

Are there are finitely many cases to check?

RE: Languages with Verifiers

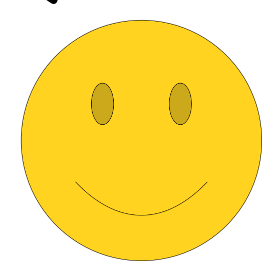
Given any string $w \in L$, could you ***prove*** that $w \in L$?



R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

Here's another approach we can take.



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REG: Problems Solvable with Finite Memory

Are there are finitely many cases to check?

RE: Languages with Verifiers

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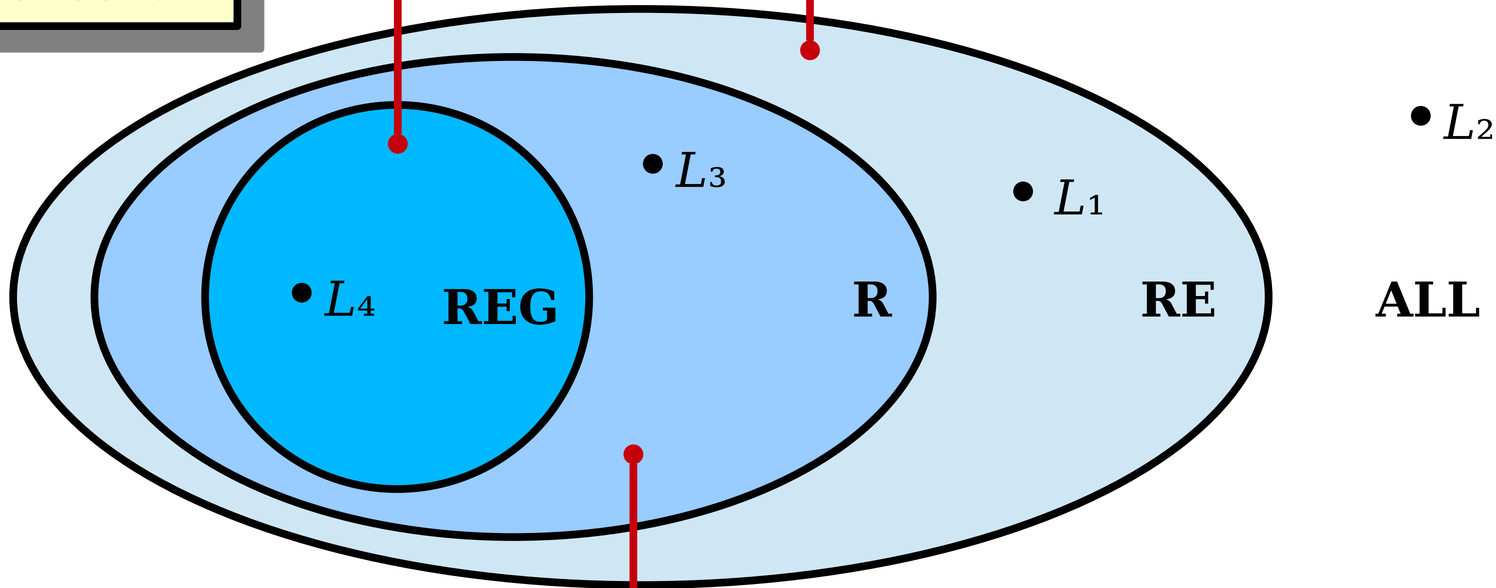
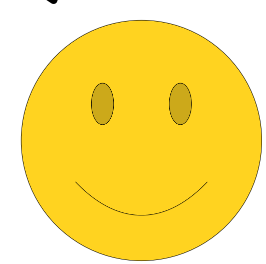


R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

How many strings are in this language?



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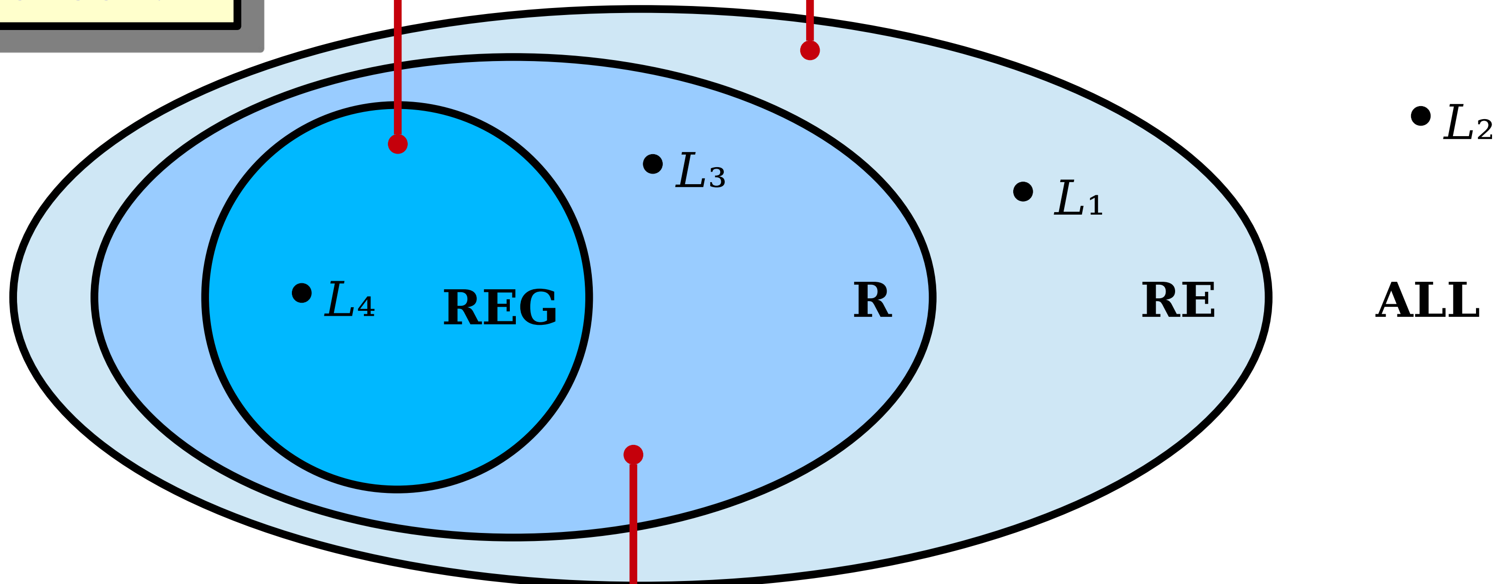

R: Languages with Deciders

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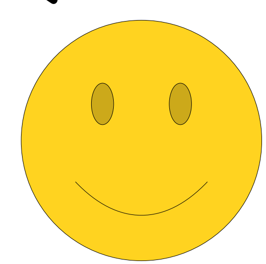
There's only 1,001 of them, corresponding to all the different choices of n we can make.



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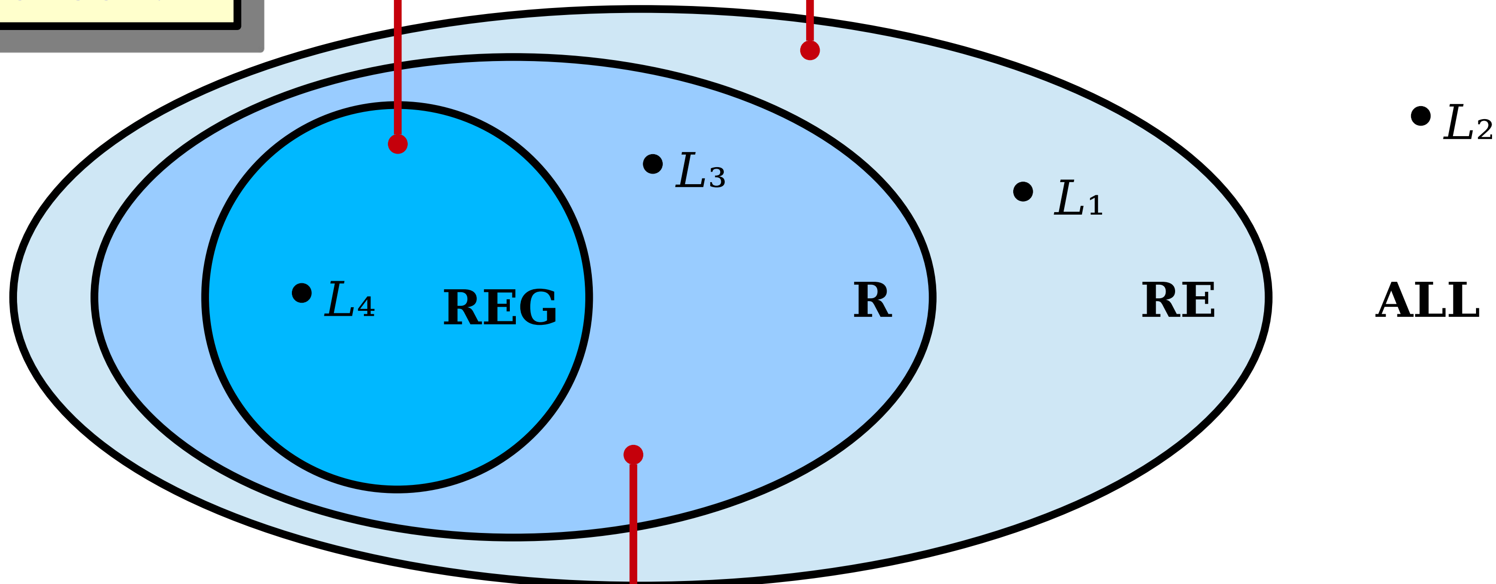

R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

As you proved on Problem Set 7, all finite languages are regular. That means that this language *has* to be regular.

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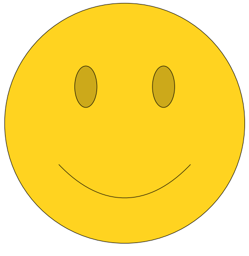

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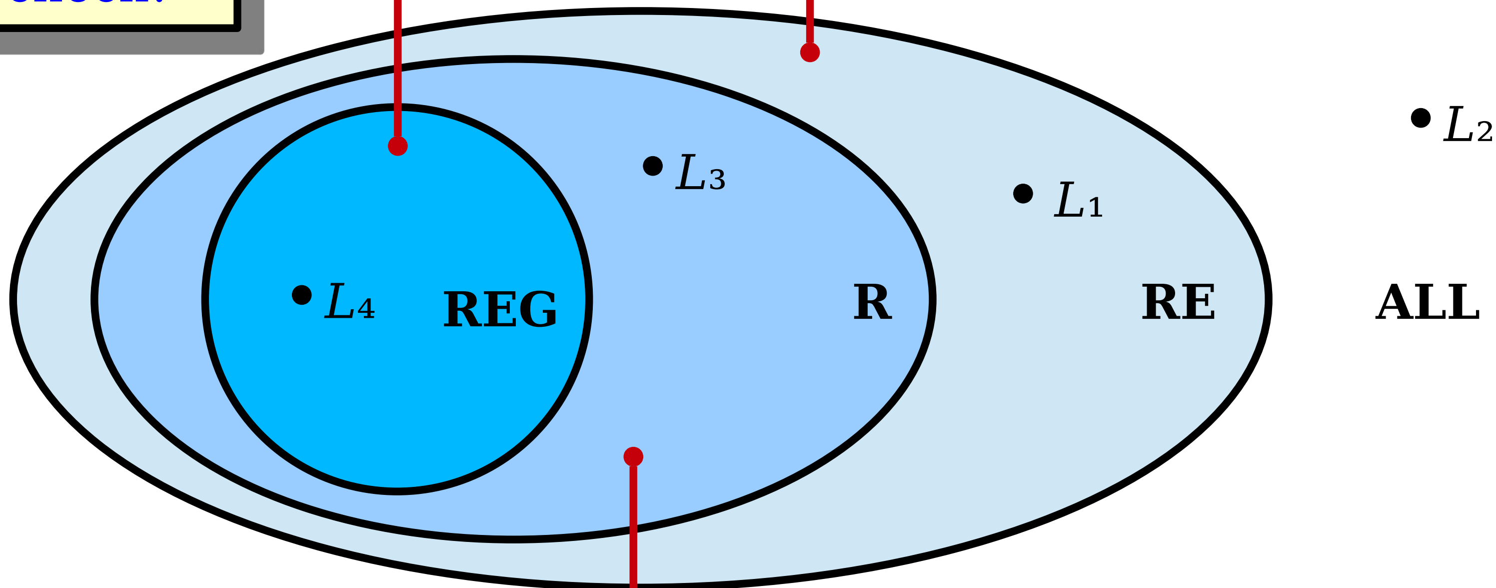

R: Languages with Deciders

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As a final option, we can think about this in terms of DFA or regex design.

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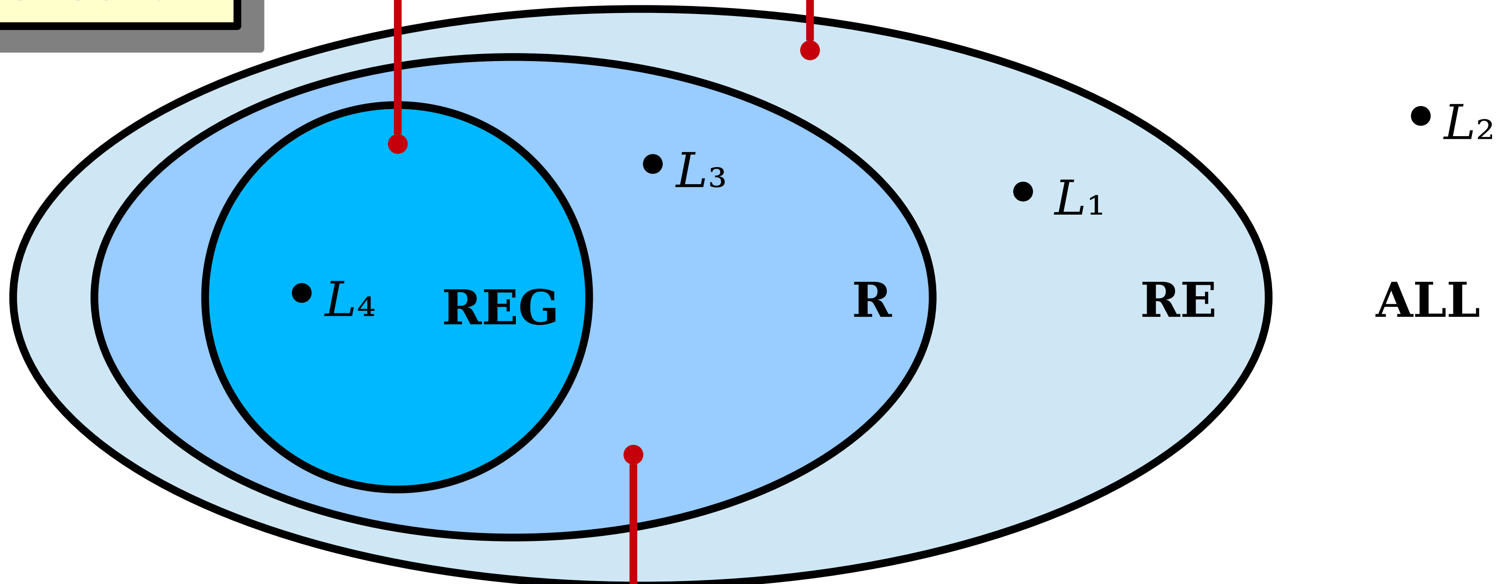
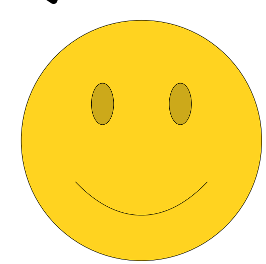


R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

You could imagine building a (huge) regex for this language:

$\varepsilon \cup ab \cup aabb \cup aaabbb \cup \dots \cup a^{1000}b^{1000}$



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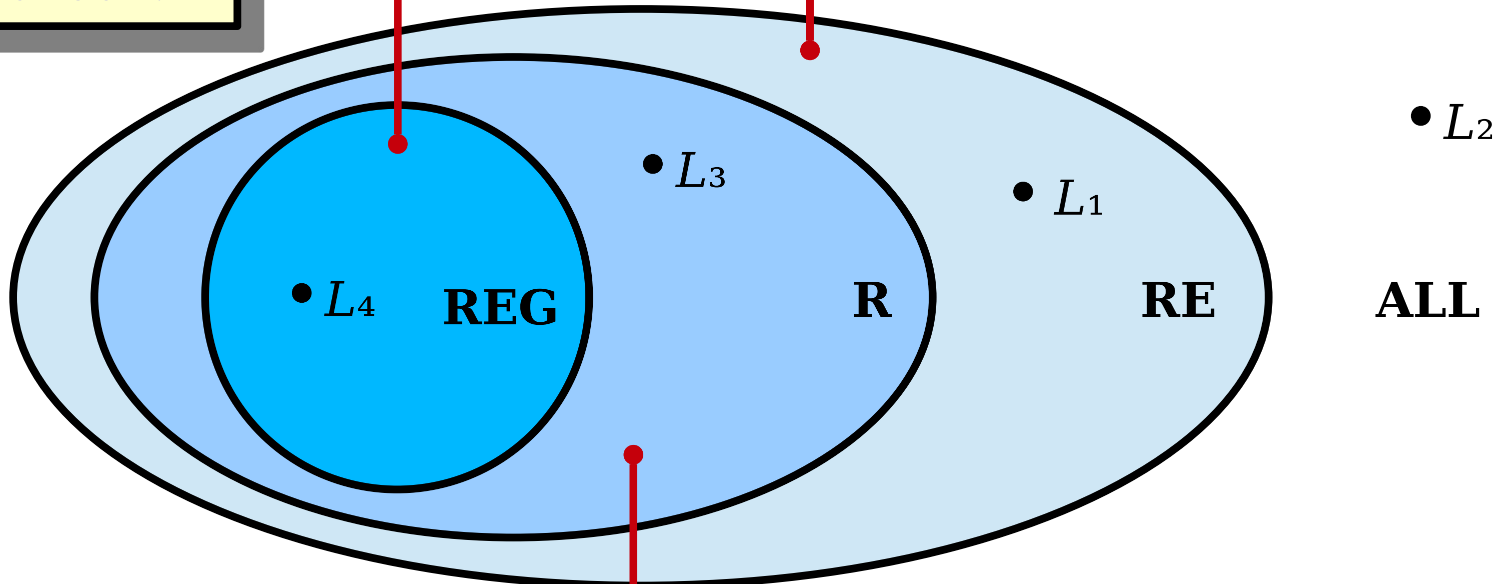

R: Languages with Deciders

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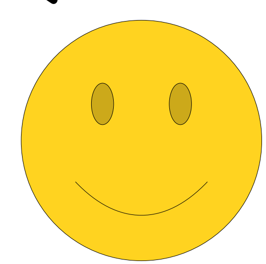
So that means that it's going to be regular.



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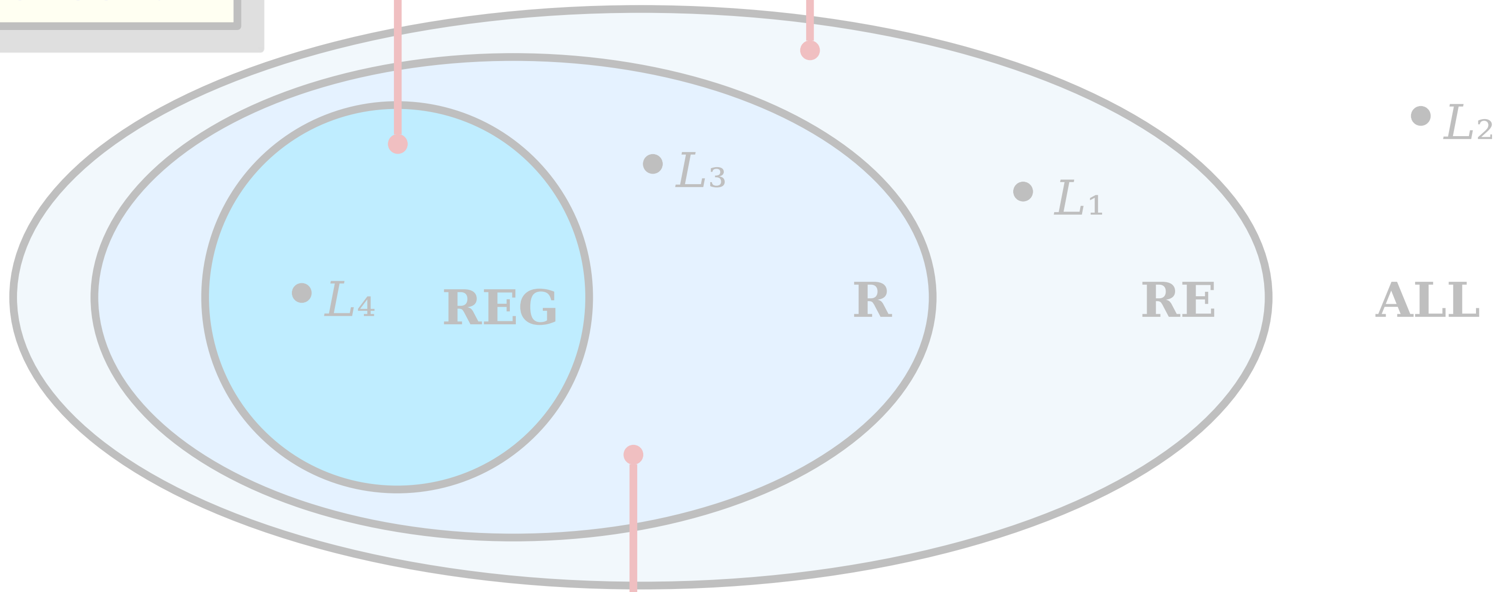

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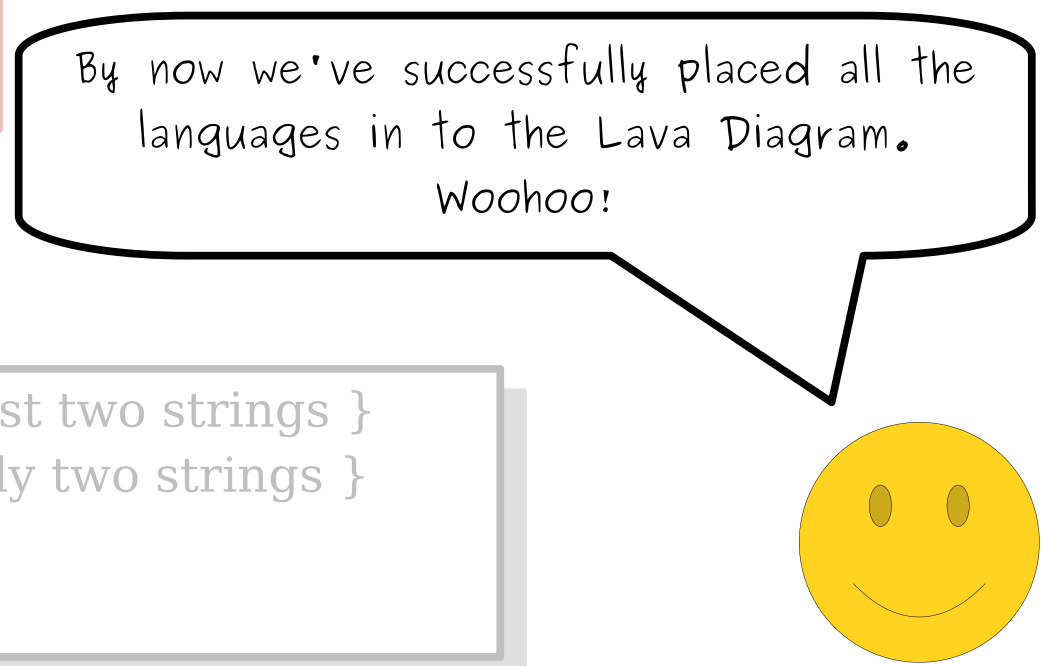

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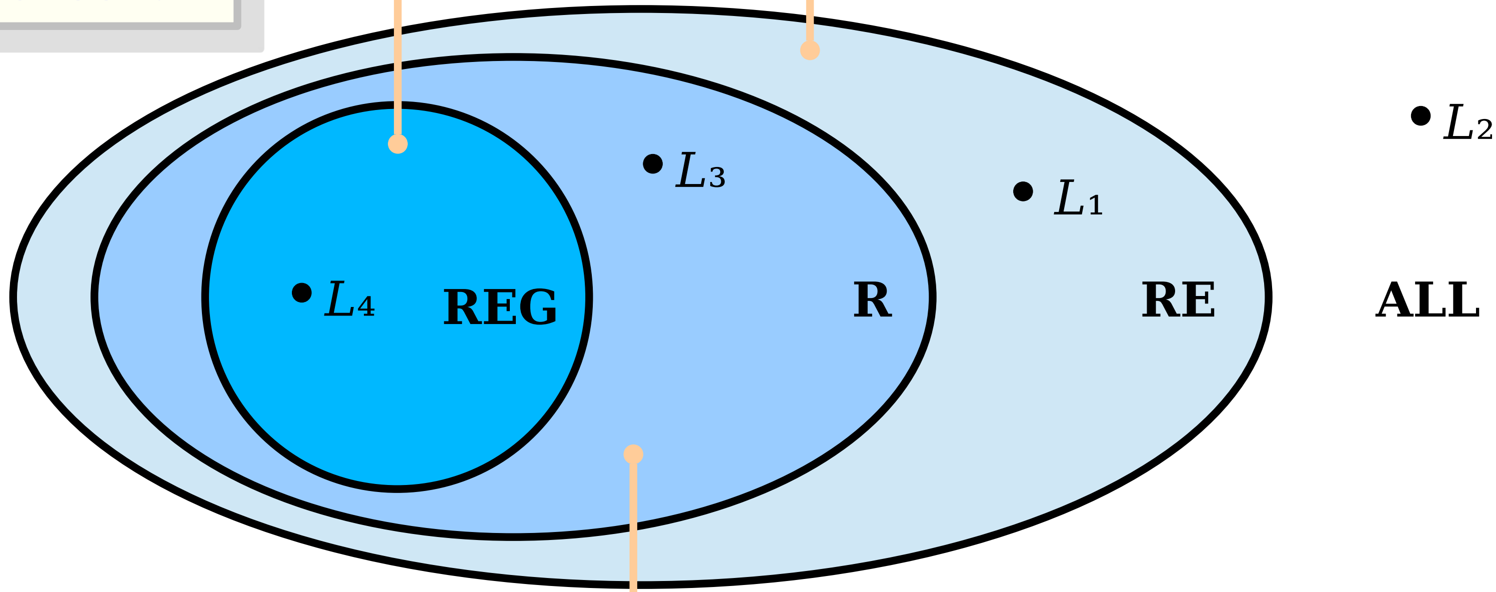

R: Languages with Deciders

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Let's do a quick recap of what all of the different regions mean and how best to think about them.

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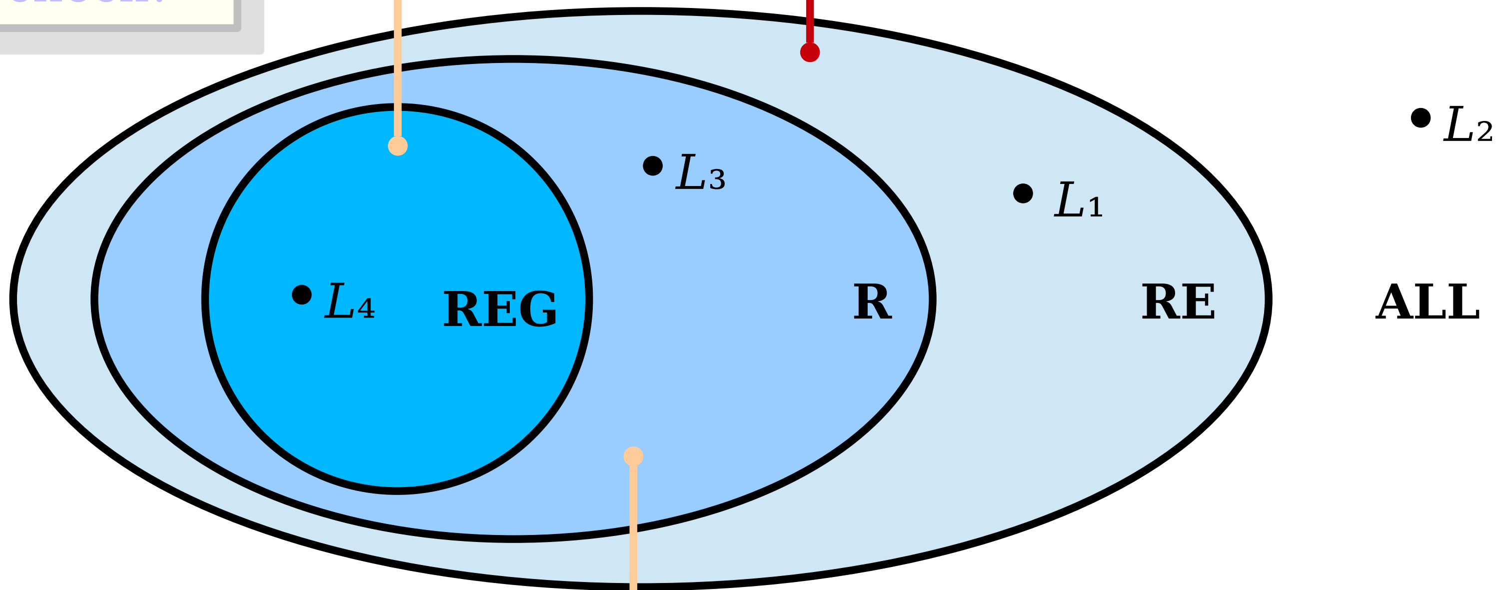

R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

First, the **RE** languages. To check whether a language is **RE**, ask yourself whether, for any string in the language, you could prove to someone else that it's in the language.

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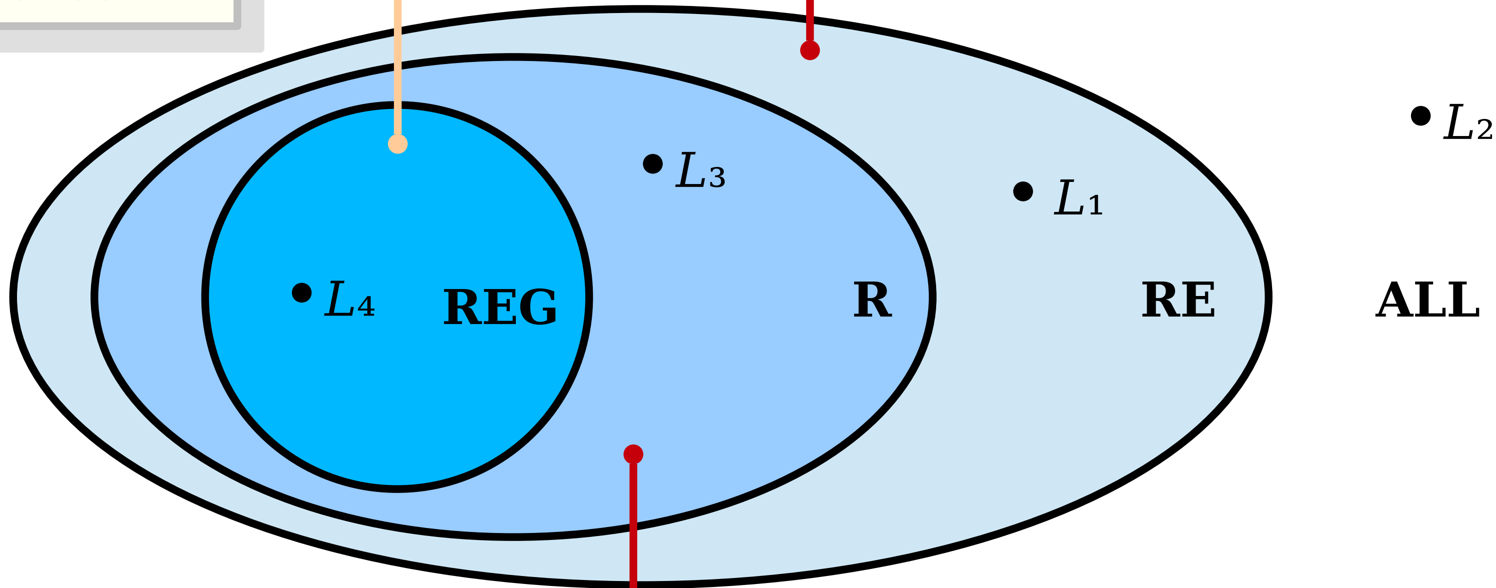

R: Languages with Deciders

In addition to the **RE** requirements, given any string $w \notin L$, could you ***prove*** that $w \notin L$?

Next, the **R** languages. If that you already know your language is in **RE**, you can figure out whether it's in **R** by asking whether, for any string not in the language, you can prove it's not in the language.

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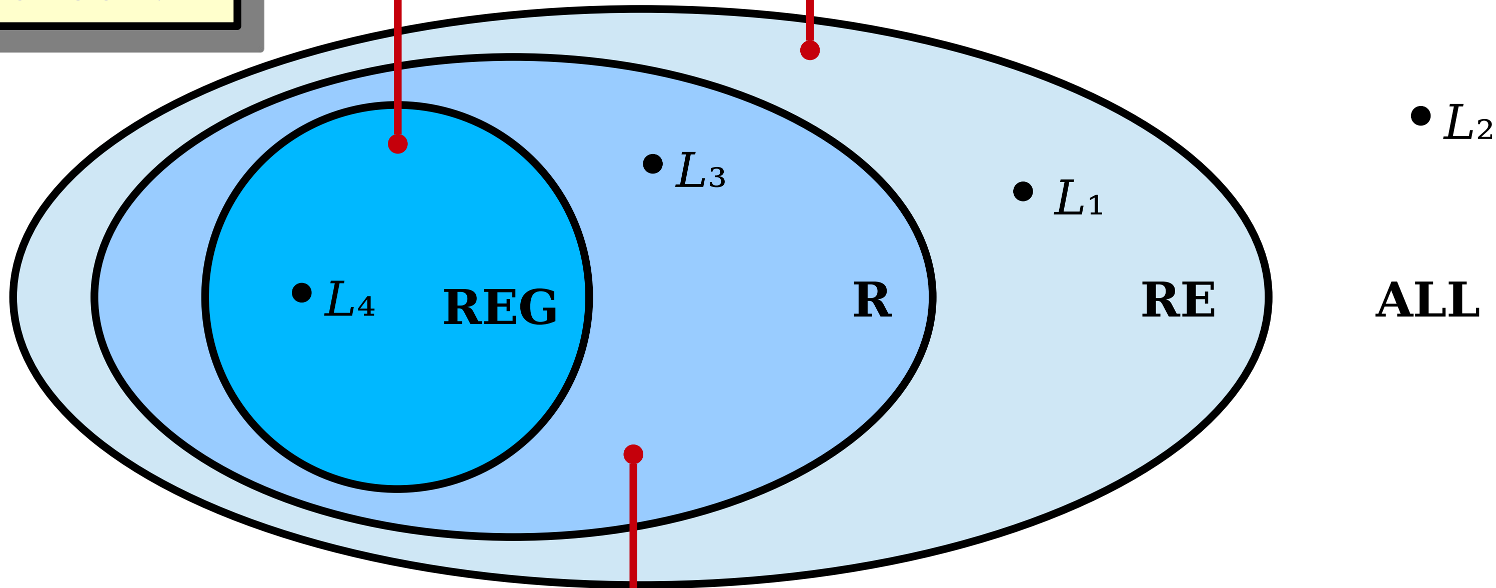

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Finally, the regular languages. Those are the ones that you can solve given only finite resources.

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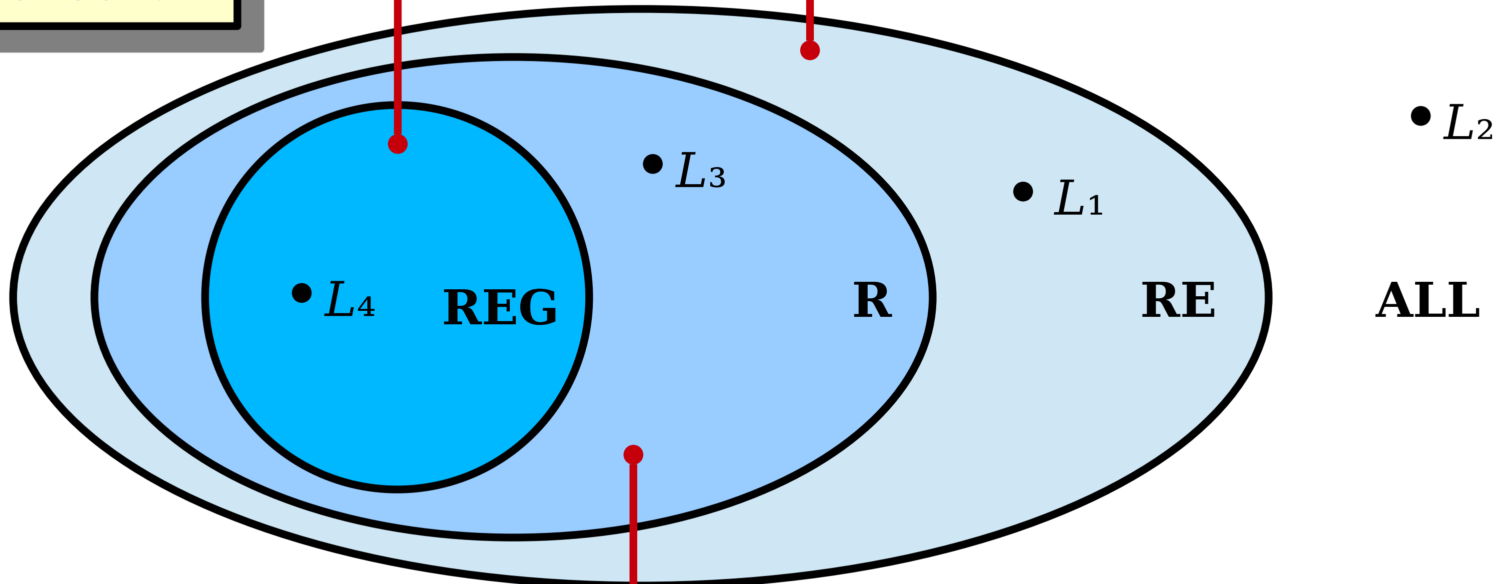

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The more that you learn about these languages, the more intuitions and nuances you'll be able to use to help guide your search.

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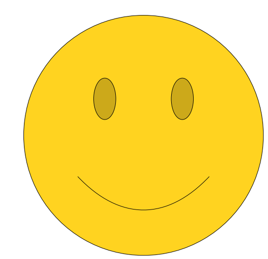

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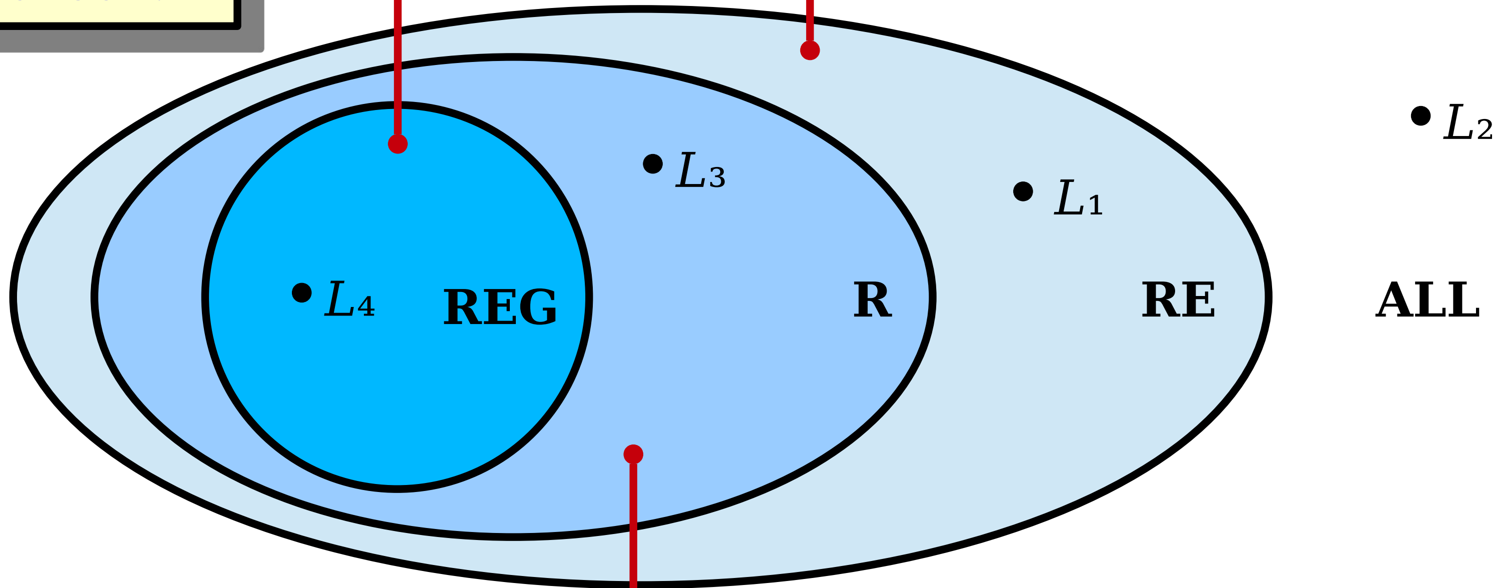

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Hopefully, this gives you a good starting point for working through Lava Diagram questions. Good luck!

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Hope this helps!
Please feel free to ask questions if you have them.

Did you find this useful? If so, let us know! We can go and make more guides like these.